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Why market shares matter: an information-based theory

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and

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Consider a duopoly market in which consumers have heterogeneous information about the quality differential q of the two goods. When firms are ignorant about q , consumers rationally believe that a firm with a high market share is likely to produce a high-quality good. As a result, firms try to signal-jam the inferences of consumers and compete for market shares beyond the level explained by short-run profit maximization. When firms know q , multiple equilibria may exist, but under a regularity condition, there is one equilibrium in which market shares signal quality, and then the market tends to be more competitive.

1. Introduction

■ In recent years economists have devoted considerable attention to the role of prices as instruments for transmitting information about market conditions or product quality to consumers. However, in many cases it seems that consumers in fact pay more attention to quantities, and it is this information that affects their consumption decisions. Examples are easy to find. Tourists usually choose relatively crowded restaurants and avoid empty ones. Readers purchase bestsellers and, unless they have seen an exceptional review, rarely buy an unknown book. Car buyers often choose from among the most popular brands. Indeed, advertising often emphasizes the fact that a brand is a top seller.

Managers, and business observers in general, also seem to be particularly concerned about quantities sold or market shares. It is, for instance, a common practice for firms to frame their strategic plans in terms of achieving market share targets. Casual evidence also suggests that in some industries the battle for market share is more aggressive than would be expected from the perspective of short-run profit maximization.

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The emphasis given to quantity signals leads one to question whether such behavior is consistent with rationality on the part of both consumers and firms. Very often, the attention consumers give to such signals is attributed to psychological or sociological concepts rather than economic ones: influence of reference groups, tendency to imitate brand image, and so on. We wish to argue that some of the patterns of consumer behavior discussed above and in the marketing literature are perfectly consistent with full economic rationality and, moreover, that they are directly related to the informational role of quantities. The interest of firms in market share then arises from the informational value attached to market shares by consumers. It may then be the case that a high market share may signal that the product is of high quality.

In this article we consider a duopoly model in which (short-lived) consumers are uncertain about their relative valuation of the varieties produced by (long-lived) firms. The uncertainty concerns the quality differential, i.e., the match between the characteristics of the varieties produced and the preferences of the consumers. Consumers, nevertheless, possess some private and imperfect information about the differential. For instance, each consumer can observe the experience of friends and family members who have used the goods previously, or might have access to an independent report on the quality of the goods. The signals received by individual consumers will typically not be perfectly correlated. This private information influences the decisions of consumers, and therefore data on aggregate sales may provide an additional source of information. A higher current market share can be interpreted by future consumers as a signal of higher relative quality and will tend to increase future demand. Due to the presence of consumers who patronize firms at random ("noise consumers"), however, market share will be an imperfect signal of the quality differential.

Our model assumes that past prices are unobserved by consumers, who must then make their quality inferences on the basis of past quantities sold, which are observed. Although extreme, this assumption captures in a stark way an important feature of practical experience. In many markets, past quantities sold are often directly observable (for example, a restaurant's popularity could be learned from a guide or just oral transmission, or a car model's sales could be learned just by observing the number of cars in the street or reading consumer reports). By contrast, consumers often have very imprecise information about the prices previous consumers have paid. Even if they remember the prices quoted in previous years, local retail price variations and secret price discounting often mean that quoted prices and transaction prices bear little resemblance to each other.

We examine two variants of the basic model. In the first, benchmark case, the firms themselves do not know how well their products accord with consumers' tastes. In such a setup, it is then impossible to signal quality. However, the firms know that future consumers will try to learn from the experience of current consumers by observing market shares. By cutting current prices, which are not observed by future consumers, the firms can attempt to fool those consumers into thinking that their product is selling well because of its high quality and not simply because it is cheap. Although consumers are not taken in, these incentives ensure that prices are set below short-run profit-maximizing levels.

In the second variant of the model, we allow for firms themselves to know the quality differential. This opens up the possibility of using (current) prices to signal quality. Of course, consumers still use the information contained in past market shares. We find that there may then be multiple equilibria but, under a regularity condition, that there is one equilibrium in which market shares are a positive signal of quality.¹

¹ Throughout the article, by market shares being a positive signal of quality we mean that the expected value of the quality differential (in favor of firm A) conditional on consumers' information is increasing in the past market shares of firm A.

However, in general there is a tension between the incentives to manipulate consumers' beliefs through prices or through quantities, which tend to have opposite effects on expected prices. Consequently, the degree of competition may turn out to be higher or lower than in the case of complete information, depending on the parameter values (for the same reason, the sequence of expected prices may be increasing or decreasing). Nevertheless, the informational role of market shares will tend to make the market more competitive whenever consumers infer high relative quality from a high market share.

In the next section we briefly review the related literature. In Sections 3 and 4 we describe and analyze the benchmark model with two periods and uninformed firms. In Section 5 we assume that firms are perfectly informed about the quality differential, and thus, prices as well as quantities convey relevant information. Concluding remarks follow.

2. Review of related literature

■ We obtain the results in the context of a linear-quadratic model with normally distributed random variables. This type of model is standard in the literature of information sharing in oligopoly. (See, for example, Clarke (1983), Vives (1984), and Galor (1985).) Our article has several connections with the literature on alternative explanations of the value of market shares (switching costs and network externalities), competition with signal-jamming, and models of prices as signals of quality.

Competition for market shares, beyond the level that could be explained by short-run profit maximization, is also the outcome of the theory of competition in markets with switching costs (see, for example, Klemperer (1987) and Beggs and Klemperer (1992)). The literature on network externalities (see, for instance, Katz and Shapiro (1986)) provides a technological explanation of the role of previous market shares in boosting current demand. Our story is complementary to this work.

There may also be sociological reasons to justify the fact that the value of a good for an individual consumer depends positively on the number of consumers who choose the same good. This is the approach taken by Becker (1991), who also states an alternative information-based motive. The present article could be understood as the formal development and analysis of the implications of the information-based reason with Bayesian-rational agents. With nonrational consumers, Smallwood and Conlisk (1979) study market dynamics with exogenous prices and where consumers' adaptive behavior is described by a rule of thumb with the property that the probability that a consumer shifts to a new brand increases in its previous market share.

Our article is also related to various models of competition with signal-jamming (see Riordan (1985), Fudenberg and Tirole (1986), and Holmström (1982)). Signal-jamming in general involves garbling relevant information received by other players in the game with a hidden action. In our model, firms attempt to signal-jam the information that consumers are trying to infer from market shares using unobservable past prices.

Most of the literature on monopoly pricing with asymmetric information on product quality has devoted itself to analyzing the circumstances under which prices can signal quality (how a high-quality firm can separate itself from one with low quality by pricing differently). In some circumstances, a high-quality monopolist will set a sequence of prices that increases over time (introductory offers), and in other circumstances the sequence will be decreasing. The first possibility is illustrated by Milgrom and Roberts (1986). If consumers repeat purchases, the monopolist sets a low initial price to induce experimentation. As consumers become more informed, the monopolist increases the price. However, when consumers purchase the good only occasionally but

a fraction of consumers are fully informed about the quality of the good, the monopolist may use a high initial price to signal high quality and decrease it over time as the fraction of informed consumers rises (Bagwell and Riordan, 1991). In our model, when firms are not informed about the quality differential, we typically obtain an increasing sequence of expected prices (introductory offers).

In a model with consumer learning that also uses a linear-normal specification, Judd and Riordan (1994) find that higher prices may signal higher quality independently of cost asymmetries, and that the monopolist has incentives to invest in quality (with the investment being unobservable to consumers) to manipulate consumer beliefs and increase demand in future periods. Thus, in contrast to our model, signal-jamming of consumer inferences is achieved via unobservable investment.

3. The model

■ In this section we describe and analyze the simplest model that captures the key elements of our theory. We also provide alternative interpretations of some of the assumptions and discuss extensions that require only small changes in the specification of the model without altering the qualitative results. In the next section we expand the analysis to allow for better-informed firms.

The basic model is as follows. Two firms, A and B (denoted by the superscript j , $j = A, B$), produce goods that are imperfect substitutes. The quantity of good j in period t is denoted by x_t^j . Variable costs are assumed to be zero or, alternatively, marginal costs are constants and identical for both firms, and prices are defined net of marginal cost.

There is a continuum of consumers who enter the market and stay there for just one period. We therefore eliminate any intertemporal aspect of the consumer's decision problem. Consumers take a utility function that is quadratic in the goods produced by the two firms and additively separable with respect to money. Consumers differ only in their information. The representative consumer's utility in the differentiated products ($x = (x^A, x^B)$) is given by

$$U(x) = [\alpha + (\beta - \gamma)q]x^A + [\alpha - (\beta - \gamma)q]x^B - \frac{1}{2}[\beta(x^A)^2 + 2\gamma x^A x^B + \beta(x^B)^2],$$

where α , β , and γ are positive parameters, with $\beta > \gamma$. Fixing β , γ represents the degree of product differentiation: the smaller it is, the more the products are differentiated (for $\gamma = 0$ the products are independent, and for $\gamma = \beta$ they are perfect substitutes).

The representative consumer's willingness to pay for variety A is $[\alpha + (\beta - \gamma)q]$ (and $[\alpha - (\beta - \gamma)q]$ for variety B). Consumers are uncertain about q and learn it only after consuming the varieties (they are "experience goods"). The parameter q , a random variable normally distributed with mean zero and variance σ_q^2 , represents the matching between product characteristics and consumer preferences. We will term q the quality differential between products A and B . As the value of q increases, the utility derived from consuming good A increases while that of consuming good B decreases.²

Before making their purchases, consumers have access to an independent test of the product. Perhaps each consumer can try the product in the store, or read the report of a formal test conducted by a particular consumer magazine. In any case, we assume

² Notice that, as is usual with linear-normal models, goods may turn into "bads" depending on the value of the random variable q . Nevertheless, by controlling the variance of q , the probability of such occurrences can be controlled.

that each consumer receives an independent noisy signal of q . More specifically, consumer i in period t observes s_{it} , $s_{it} = q + \epsilon_{it}$, where ϵ_{it} is an independent random variable, normally distributed with mean zero and variance σ_ϵ^2 .³ By analogy with the Strong Law of Large Numbers, we make the convention that the average signal received by consumers of generation t equal q almost surely.

Let I_{it} denote the information available to consumer i in period t . Expected utility maximization and price-taking behavior given prices (p_t^A, p_t^B) imply

$$\begin{aligned} x_{it}^A &= a + E(q|I_{it}) - bp_t^A + cp_t^B \\ x_{it}^B &= a - E(q|I_{it}) - bp_t^B + cp_t^A, \end{aligned}$$

where $a = \alpha/(\beta + \gamma)$, $b = \beta/(\beta^2 - \gamma^2)$, and $c = \gamma/(\beta^2 - \gamma^2)$.

Besides the above “rational” consumers, there are some other consumers whose purchasing decisions are made at random. The aggregate purchases of good j in period t by the set of random consumers is denoted by u_t^j , a normally distributed variable with mean zero and variance σ_u^2 (uncorrelated with all other random variables). It is assumed that rational consumers, indexed by i , are distributed in the unit interval $[0, 1]$, endowed with the Lebesgue measure. It follows that aggregate demands in period t are

$$\begin{aligned} x_t^A &= a + \eta_t - bp_t^A + cp_t^B + u_t^A \\ x_t^B &= a - \eta_t - bp_t^B + cp_t^A + u_t^B, \end{aligned}$$

where $\eta_t = \int_0^1 E(q|I_{it}) di$ is the aggregate belief of rational consumers in period t .

The role of the random consumers is analogous to that of the noise traders in the finance literature (see, for example, Kyle (1985)). In their absence, by observing aggregate sales and correctly forecasting the prices of a single period, the realization of q can be inferred with certainty. With these random consumers, aggregate sales can only be a noisy signal of q .⁴

There are two periods, $t = 0, 1$. At the beginning of period 0, the random variable q is realized and stays a constant thereafter. Firms simultaneously quote prices $p_0 = (p_0^A, p_0^B)$, with no information about q . Consumer i observes s_{i0} . Thus, the information vector of consumer i in period 0 is $I_{i0} = \{s_{i0}, p_0\}$. On the basis of such information, consumers choose quantities demanded and the variables u_0^A, u_0^B are realized. Finally, firms produce $x_0 = (x_0^A, x_0^B)$.

In period 1, firms set prices $p_1 = (p_1^A, p_1^B)$ having observed the quantities sold in period 0. The information vector of firm j is therefore $I_1 = \{x_0, p_0\}$. Consumers have observed the quantities sold in period 0 but not the prices set by firms. The information vector of consumer i in period 1 is thus $I_{i1} = \{s_{i1}, x_0, p_1\}$. Similarly, demand and production are realized as before.

³ Alternatively, we could interpret the signal as the outcome of word-of-mouth communication. Assume that individual consumers have positively but imperfectly correlated preferences. In particular, the individual valuation of the quality differential has a common component and an idiosyncratic one, $q_{it} = q + \epsilon_{it}$, and each consumer has access to the experience of a previous consumer selected randomly. Since individual valuations are imperfectly correlated across consumers, observing the experience of a friend provides only a noisy signal of the own valuation.

⁴ These consumers can be interpreted literally as irrational agents, in the sense that they make their purchases without taking into account their information on the quality differential and prices. Alternatively, we could have assumed that there is some uncertainty about the distribution of consumers. In this case, aggregate sales would also be a noisy signal of the quality differential. With our formulation we obtain a specific, and very simple, reduced form.

Two remarks on the information structure can be made at this point. First, firms receive no private information about q . In this context, quantities, not prices, will convey information, and hence the role of this new vehicle of information transmission can be more clearly understood. This assumption will be relaxed in Section 5, where both prices and quantities provide useful information.

Second, consumers observe only current prices and past quantities, not past prices. At a descriptive level, in many markets past quantities are more easily observable than past prices. One may easily observe which types of cars previous consumers decided to purchase in the past, or the brand of the most popular camera or videotape recorder, either through direct observation of a relatively large sample or through consumer reports. However, it is not so easy to learn which prices were actually paid. Perhaps consumers can remember posted prices, but it is unlikely that they have observed actual transaction prices (whether previous consumers had access, for instance, to store discounts).⁵

Firms maximize the expected present value of profits with discount factor δ . The notion of equilibrium we use is perfect Bayesian. That is, all agents maximize their expected payoffs at any point in time given the beliefs they have, and beliefs are consistent in the Bayesian sense with strategies (that is, beliefs are obtained from equilibrium strategies and observations using Bayes' rule).

As usual, there are multiple equilibria depending on the assumptions made on conjectures off the equilibrium path (for which Bayesian updating has no bite). We shall concentrate on equilibria with the property that consumers do not infer any information from prices: perfect Bayesian equilibria with signal-free prices. In equilibrium, consumers' conjectures on past prices are correct, and therefore the information of consumers is better than that of firms. Thus, observed prices do not provide any additional information. However, if a firm deviates from equilibrium pricing, it will have different information from future consumers, and in this case prices may potentially reveal some of this differential information. We abstract from this possibility by assuming that the beliefs of consumers are constant, that is, unchanged, when they observe an out-of-equilibrium price (constant beliefs out of the equilibrium path).⁶ Moreover, in Section 5 we deal with the case in which firms are better informed than consumers on the realization of q . In such a context the signalling role of prices will be analyzed.

4. Equilibrium characterization

■ In order to solve for the perfect Bayesian equilibria with information-free prices, we have to take care of the updating problem of consumers and firms and the optimization problem of the latter.

□ **Updating beliefs: consumers.** In period 0 the only source of information of consumers on q is their private signal. They also observe the (equilibrium) prices p_0 , but they are not informative (firms do not have any private information about q). Given the normality assumption about all random variables, consumer i 's beliefs are a linear function of his private signal:⁷

⁵ Readers worried about the nonobservability of past prices should consider the following. In terms of the incentives of consumers to acquire information, it is important to notice that prices are not random variables but decisions made by firms. In equilibrium, consumers are able to compute these prices correctly. Thus, if consumers are to pay an arbitrarily small cost to observe past prices, they will choose not to do so; or if only a noisy signal of past prices is observed (with arbitrary precision), rational consumers need not use such information, since they are able to forecast equilibrium prices.

⁶ A similar assumption is made in Hart and Tirole (1990).

⁷ We will write τ_y for the precision of the random variable y (i.e., $\tau_y = 1/\sigma_y^2$).

$$E(q|I_{i0}) = m_0 s_{i0},$$

where $m_0 = \tau_\epsilon / (\tau_\epsilon + \tau_q)$. Aggregate beliefs (according to our convention on the average signal) are given by

$$\eta_0 = \int_0^1 E(q|I_{i0}) di = m_0 q.$$

Demand in period 0 is then:

$$\begin{aligned} x_0^A &= a + m_0 q - bp_0^A + cp_0^B + u_0^A \\ x_0^B &= a - m_0 q - bp_0^B + cp_0^A + u_0^B. \end{aligned}$$

In period 1, consumers observe the quantity vector x_0 and current prices p_1 , and they receive signals about q . Since they do not observe period 0 prices, they need to have conjectures about them, denoted by p_0^e , to interpret the quantities sold in period 0 as (noisy) signals of q . Consumers have also conjectures about the strategies used by firms in period 1: $p_1^i(X_0, p_0)$. In equilibrium the conjectures are correct. What happens when consumers observe, given x_0 , an out-of-equilibrium first-period price? As indicated above, we assume that consumer beliefs stay constant; that is, out-of-equilibrium first-period prices are not informative.

The information vector of consumer i in period 1 is therefore $I_{i1} = \{s_{i1}, x_0, p_1\}$. Given variables y^A and y^B , let $\Delta y = y^A - y^B$. With conjectured prices p_0^e , consumer i 's estimate of q is a weighted average of his private signal and the difference in quantities sold in the previous period Δx_0 (the "market share"). A consumer observing a higher signal or a higher market share for firm A will think that q is high. More precisely,

$$E(q|I_{i1}) = m_1 s_{i1} + n_1^0 z_0^e,$$

where

$$m_1 = \tau_\epsilon / \tau_{i1}, \quad n_1^0 = 2\tau_u m_0^2 / \tau_{i1}, \quad \tau_{i1} = \tau_\epsilon + \tau_q + 2\tau_u m_0^2$$

and

$$z_0^e = [\Delta x_0 + (b + c)\Delta p_0^e] / 2m_0.$$

Consumer i uses his private information and the statistic z_0^e of public information to estimate q . Note that the information in market shares increases the precision of consumers in the estimation of q by $2\tau_u m_0^2$. Given the assumed symmetric structure of preferences, only the difference in quantities Δx_0 matters to form z_0^e . Further, consumers use their conjectures on unobservable period 0 prices to "read" market shares, and this allows firms the possibility of manipulating period 1 consumer beliefs. If firms set prices p_0 , the period 0 market share is given by $\Delta x_0 = 2m_0 q - (b + c)\Delta p_0 + \Delta u_0$. Firms in period 0 can jam the signals (noisy market shares) that consumers receive. Indeed, it is easily seen that

$$z_0^e = q + \frac{1}{2m_0} (\Delta u_0 - (b + c)(\Delta p_0 - \Delta p_0^e)).$$

Consumers' aggregate belief equals

$$\eta_1 = \int_0^1 E(q|I_{i1}) di = m_1q + n_1^0 \left\{ q + \frac{1}{2m_0} (\Delta u_0 - (b + c)(\Delta p_0 - \Delta p_0^e)) \right\}.$$

It is clear that beliefs held by consumers can be manipulated by firms. In fact

$$\frac{\partial \eta_1}{\partial p_0^A} = -\frac{\partial \eta_1}{\partial p_0^B} = -(b + c) \frac{n_1^0}{2m_0}.$$

By undercutting its price, a firm tends to increase its market share. Since price expectations are exogenous, a larger market share is interpreted by consumers as a signal of higher quality. Note that $b + c = (\beta - \gamma)^{-1}$, and therefore an increase in the degree of product substitutability (a decrease in $\beta - \gamma$) increases the effectiveness of the manipulation by firms.

In equilibrium, however, consumers' conjectures are correct, and therefore beliefs are

$$\eta_1^* = m_1q + n_1^0 z_0,$$

with $z_0 = q + (1/2m_0) \Delta u_0$.

□ **Updating beliefs: firms.** In each period, firms will try to forecast consumers' aggregate beliefs, η_t . Thus we can define firm j 's beliefs on η_t as $\theta_t = E(\eta_t|I_t)$. Notice that both firms have the same information, so a firm cannot manipulate the beliefs of the other firm.

In equilibrium, firms' beliefs are given by $\theta_t^* = E(\eta_t^*|I_t)$. This is easily seen to be equal to $E(q|I_t)$. The beliefs of firms about the beliefs of consumers on q just equal firms' beliefs on q . We have then $\theta_0 = \theta_0^* = 0$, and $\theta_1 = E(\eta_1|z_0) = m_1E(q|z_0) + n_1^0 z_0^e$.

Firms can manipulate consumers' beliefs, and consequently their expectation about the intercept of the demand function in period 1 depends on their pricing policy:

$$\frac{\partial \theta_1}{\partial p_0^A} = -\frac{\partial \theta_1}{\partial p_0^B} = \frac{\partial \eta_1}{\partial p_0^A} = -\frac{\partial \eta_1}{\partial p_0^B} = -(b + c) \frac{n_1^0}{2m_0}.$$

□ **Equilibrium pricing.** To compute the equilibrium price sequence, we must proceed backward. Given period 0 realized market shares, prices p_0 and consumers' conjectures p_0^e , in period 1, firm A 's optimization problem consists of choosing p_1^A in order to maximize

$$\Pi_1^A = (a + \theta_1 - bp_1^A + cp_1^B)p_1^A.$$

Firm B solves a similar problem. The equilibrium price vector is

$$p_1^A = \frac{a}{2b - c} + \frac{\theta_1}{2b + c}$$

$$p_1^B = \frac{a}{2b - c} - \frac{\theta_1}{2b + c}.$$

Given these prices, firm A 's expected demand is given by

$$E(x_1^A | I_1) = b \left(\frac{a}{2b - c} + \frac{\theta_1}{2b + c} \right).$$

In period 0 the optimization problem of the firm is slightly more complicated. It must choose p_0^A in order to maximize

$$\Pi_0^A = (a + \theta_0 - bp_0^A + cp_0^B)p_0^A + \delta E\{p_1^A x_1^A | I_0\}.$$

Notice that p_0^A affects θ_1 and thus alters expected demand in the next period, as well as expected prices. Firm *B* again solves a similar problem. Given that $\theta_0 = E\{\theta_0^* | I_0\} = 0$ and $\partial\theta_1/\partial p_0^A = -\partial\theta_1/\partial p_0^B$, equilibrium prices are equal to⁸

$$p_0^B = p_0^A = \left\{ 1 + \frac{2b\delta}{4b^2 - c^2} \frac{\partial\theta_1}{\partial p_0^A} \right\} \frac{a}{2b - c}.$$

Since the effect of p_0^A on θ_1 is negative, the price charged in period 0 is lower than the expected (conditional to I_0) price that will be set in period 1. The intuition is straightforward. In period 0, firms have an additional incentive to cut prices. By reducing its price below the level that maximizes period 0 profits, a firm increases its market share by positively influencing beliefs held by future consumers on the relative quality of its product. This increases future demand and future profits. The following proposition summarizes the previous discussion.

Proposition 1. In equilibrium,

$$p_0^j = \left\{ 1 - \frac{b\delta}{4b^2 - c^2} (b + c) \frac{n_1^0}{m_0} \right\} \frac{a}{2b - c}, \quad j = A, B,$$

and

$$p_1^j = \frac{a}{2b - c} + \lambda^j \frac{\theta_1^*}{2b + c}, \quad \text{with } \lambda^A = 1, \lambda^B = -1.$$

Period 0 prices are below the one-shot level $a/(2b - c)$ and may even be negative (that is, below marginal cost). The incentive to reduce prices in period 0 increases with the discount factor δ (weight to future profits); with the precision of demand noise τ_u (precision of market shares as signals of q), since consumers pay more attention then to market shares; and with the degree of substitutability of the products γ , since a deviation from equilibrium pricing is more effective in boosting market share when the products are better substitutes.⁹ The effects of changes in the precision of consumer signals τ_e are not monotonic. Indeed, for τ_e equal to zero, or τ_e equal to infinity, there is no incentive to manipulate consumers' beliefs, either because market shares are not informative or because consumers are perfectly informed. For intermediate values of the parameter it does pay to try to influence consumers' beliefs. The incentive to manipulate is decreasing with respect to τ_q . Indeed, when there is no uncertainty there is no incentive to manipulate beliefs.

⁸ In order to ensure concavity of Π_0^A with respect to p_0^A , the following inequality must hold: $\delta[(b + c)/(2b + c)]^2(n_1^0/2m_0)^2 < 1$. If this inequality does not hold, then there is no symmetric pure-strategy equilibrium. Thus, existence is guaranteed when the discount factor is not too large, goods are not close substitutes, or consumers do not put too much weight on past market shares in updating their beliefs.

⁹ The result for δ is obvious; n_1^0/m_0 is increasing in τ_u and $b(b + c)/(4b^2 - c^2) = \beta(\beta + \gamma)/(4\beta^2 - \gamma^2)$.

The responsiveness of period 1 prices to public information on q , $(2b + c)^{-1}$, is decreasing with the degree of substitutability of products ($2b + c = (2\beta + \gamma)/(\beta^2 - \gamma^2)$, which is increasing in γ). Indeed, when γ approaches β , the utility translation of the quality differential, $(\beta - \gamma)q$, goes to zero.

Thus, in this model, competition for market shares leads to a more aggressive price behavior in the first period, while in the second period prices coincide with those of the one-shot game. Thus, on average the market becomes more competitive. This contrasts with the theory according to which competition for market shares arises due to switching costs (see Klemperer (1987, 1992)). In this case the price sequence is also increasing, but markets tend to be less competitive because firms' incentives to exploit locked-in consumers tend to dominate their incentives to attract new customers.

The typical pattern of low initial prices can be related to the literature on introductory offers (Milgrom and Roberts, 1986). In their theory, a monopolist is willing to set a low initial price to induce consumers to experiment, learn about the high quality of the product, and repeat purchases in the future at monopoly prices.¹⁰ In our model, oligopolists set low initial prices for a different reason. Consumers do not repeat purchases, but their actions are observed by future consumers.

Such a mechanism creates market share inertia, as in the literature on switching costs or network externalities. If a firm's current market share increases, say because of a higher realization of u , future consumers positively change their opinion on the relative quality of the product and increase their demand. Consequently, the firm's future expected demand increases, which may lead the firm to raise its price by relatively little so as to increase further its expected future demand.

5. Informed firms

■ In the benchmark model we assumed that firms do not receive any private information about the realization of q . In this section we examine how robust the results obtained are to changes in this assumption.

If firms receive some private information about q , then their actions may reveal such information. Thus, in this case, prices as well as quantities may have some informational content. Further, if firms' private signals are not perfectly correlated, then prices will signal information not only to consumers but also to the rival firm (see, for example, Mailath (1989)). We avoid this potential complication here by making the extreme assumption that firms are perfectly informed about the realization of q from the very beginning.

The reason why a specific vector of quantities (market shares) does not reveal all the information perfectly in real markets (in our model because of random consumers) also applies to prices: they are usually noisy signals (for example, if firms' marginal costs are random and not observed by consumers, then quoted prices will only be a noisy signal of q).¹¹ This is the general view taken in this section.

¹⁰ In both Milgrom and Roberts (1986) and our model, the equilibrium price sequence increases over time. Also, in their article if the marginal cost of producing a high-quality good is not higher than that of a low-quality good, then, as in our model, the first-period price is lower than under full information. However, they also allow for a positive marginal cost difference of producing high- and low-quality goods, in which case the first-period price can be higher than under full information.

¹¹ Further, even when firms are perfectly informed about the realization of q , prices may not completely reveal this information if equilibrium strategies are not an invertible function of q (pooling equilibria). In our model, since the support of q is unbounded, there cannot be equilibria with prices independent of q (completely pooling), but they might be, for instance, a step function of q (semipooling). We shall not pursue this further here.

We assume that consumers observe only retail prices. Suppose that there is a continuum of retailers. Retailer k of variety j in period t has a distribution cost ϕ_{kt}^j , where ϕ_{kt}^j is a random variable, normally distributed with zero mean (for simplicity), variance σ_ϕ^2 , and uncorrelated across retailers. Each consumer visits a single retailer for each good. The cost of a specific retailer is only known to suppliers. Hence, suppliers will optimally require retailers to set a break-even price.

In this context, the retail price of good j observed by a consumer patronizing retailer k is

$$p_{kt}^j = p_t^j + \phi_{kt}^j.$$

Since the (average) aggregate retail cost will be zero (according to our convention on the average of a continuum of identically and independently distributed random variables), the average price of good j will equal p_t^j . For notational purposes it is convenient to think that there is a retailer per consumer.¹²

We maintain the same assumptions on demand and distributions of random variables as before. The information vectors for consumer i and a generic firm are

Consumer: $I_{i0} = \{s_{i0}, p_{i0}\} \quad I_{i1} = \{s_{i1}, p_{i1}, x_0\},$

Firm: $I_0 = \{q\} \quad I_1 = \{q, x_0, p_0\}.$

We shall concentrate our attention on linear equilibria. Linear equilibria will necessarily obtain whenever consumers use linear updating rules. Period 0 price strategies will have the form

$$p_0^A = f_0 + g_0q$$

$$p_0^B = f_0 - g_0q,$$

and period 1 strategies will have the form

$$p_1^A = f_1 + g_1q + h_1\Delta u_0$$

$$p_1^B = f_1 - g_1q - h_1\Delta u_0.$$

The information $\{x_0, p_0\}$ is summarized in the differential noise trading Δu_0 . Firms observe the realization of q as well as past prices. In consequence, in period 1, by observing quantities x_0 firms can infer Δu_0 . Indeed, at the proposed equilibrium, consumer i 's estimate of q in period 0, $E(q|I_{i0})$, will be a linear function of s_{i1} and Δp_{i0} . According to our convention on the average of independent random variables, the average estimate, η_0 , will be a function of q and Δp_0 . It follows that from the demand functions, Δu_0 can be inferred with knowledge of q , Δp_0 , and Δx_0 .

We characterize linear perfect Bayesian equilibria, solving as usual the problems of updating consumers' beliefs and firms' optimization. Once we have characterized the equilibria, we ask under what conditions (1) market shares provide a positive signal of q and (2) expected prices (before q is realized) are below the case in which market shares are not observable or, alternatively, below the complete-information case. Since prices depend on the consumers' rule for updating beliefs, which in turn depends on conjectured price strategies, typically there will be multiple equilibria, even in the linear

¹² More realistically, we could think that there is a continuum of consumers per retailer in the $[0, 1]$ interval (consumers being uniformly distributed now on the unit square).

class. Thus, it is not possible in general to give an unambiguous answer to the questions posed. We present now the sketch of the argument. A full development and explicit expressions for all parameter values are given in the Appendix.

Proceeding similarly as before but now taking into account that prices are also informative, and that consumers in period 0 conjecture that producer prices will take the posited linear form, we find that consumer i 's (Bayesian) estimate of q in period 0 is given by a weighted average of his signal and the retail price differential he observes:¹³

$$E(q|I_{i0}) = m_0 s_{i0} + v_0 \Delta p_{i0}.$$

In period 1 consumers observe also the market shares of period 0 and update accordingly (taking into account that producer prices will take the linear form). The result is that consumer i 's (Bayesian) estimate of q in period 1 depends also on Δx_0 :

$$E(q|I_{i1}) = m_1 s_{i1} + v_1 \Delta p_{i1} + n_1 \Delta x_0.$$

Prices p_i are a positive signal of quality whenever v_i is positive, that is, when the estimated quality differential $E(q|I_{ii})$ increases with Δp_{ii} . This happens if and only if g_i is positive. Market shares are a positive signal of quality whenever n_1 is positive, that is, whenever $E(q|I_{i1})$ increases with Δx_0 . This occurs only if g_0 is not too high, either negative or not too positive (that is, if p_0^A does not increase very sharply with q). Note that signalling of quality happens with independence of any cost asymmetry.

As in the previous section, the existence of pure strategy equilibria requires a parameter restriction.¹⁴ Under this condition, we can show that although the sign of g_0 is undetermined in general, $g_1 > 0$ always, and there is an equilibrium in which $n_1 > 0$.

Proposition 2. Under a regularity condition, there exists an equilibrium (with linear strategies) where market shares are a positive signal of q , i.e., with $n_1 > 0$. Also, in any equilibrium, $g_1 > 0$.

Proof. See the Appendix.

At least for some parameter values, there exist equilibria with negative n_1 . This happens, for example, when prices reveal very little information, products are close substitutes, and past quantities are very informative. More specifically, take $\tau_\phi = 0$ (which implies $v_0 = v_1 = 0$), and take the limit as τ_ϵ goes to 0, τ_u goes to infinity, and c goes to b . In this case, at the limit $n_1^2 = 27/(16\delta)$, which has a negative solution that satisfies the second-order condition.¹⁵

The intuition is as follows. If $\tau_\phi = 0$, τ_u is large, and τ_ϵ is small, consumers will pay a lot of attention to market shares but may conjecture that a low market share is a signal of high quality. This conjecture will turn out to be self-fulfilling. The firm's short-run profit-maximizing price increases with its product quality. Therefore, a firm with high quality is more willing to set a very high price than a low-quality firm is. If the first-period strategy is a sufficiently steep function of q , market shares will turn out to be a negative function of q , which makes the initial conjecture rational.

¹³ Without loss of generality, denote the retailer patronized by consumer i by i .

¹⁴ To ensure that total discounted profits Π_0^A are concave in p_0^A . Throughout this section we assume that the following regularity condition holds: $-(2b - v_0) + 2\delta(b - v_1)[n_1(b + c)/(2b + c - 3v_1)]^2 < 0$. This is always true for δ not too large.

¹⁵ In the case $\tau_\phi = 0$, consumers cannot infer any information from prices, but firms may still react to their information about the quality differential, since their demand intercept is a function of q . Hence, even when prices are completely uninformative, the equilibrium behavior is substantially different from the one described in Section 4.

In order to study the effect of prices and quantities as signals of quality on the degree of competition, it is worth noting first that the expected price differential for any given q , $E(\Delta p_t | q) = 2g_t q$, $t = 1, 2$, is positive when firm A has the quality advantage ($q > 0$) if and only if prices are a positive signal of quality (that is, $g_t > 0$). Recall that in our model $g_1 > 0$ always, but the sign of g_0 is undetermined in general.

The questions we want to address are whether period 0 prices will on average be below or above the complete-information case (that is, whether f_0 is lower or higher than $a/(2b - c)$), and above or below the level that would result if market shares were not observable. In equilibrium we have that

$$f_0 = \frac{a}{2b - c - v_0} \left\{ 1 - \frac{2\delta(b - v_1)(b + c)n_1}{(2b + c - 3v_1)(2b - c - v_1)} \right\}.$$

This equation indicates that there is a tension between the role of prices and quantities as a vehicle to transmit information, and such a tension is reflected in equilibrium pricing. If market shares provide a positive significant signal of quality, i.e., n_1 is positive and large, and if prices are not very informative, i.e., v_0 small, then f_0 will be below $a/(2b - c)$. However, if the informational role of market shares is not significant (n_1 close to zero) but prices are a positive significant signal of quality (v_0 positive and large), then f_0 will be above $a/(2b - c)$.

Consider some extreme examples to illustrate these possibilities. In Example 1, consumers cannot infer any information from prices (that is, $\tau_\phi = 0$). In Example 2, we suppose that τ_ϕ is arbitrarily large. In this case consumers will disregard all other sources of information, and thus the intertemporal link will be lost.

Example 1. Let $\tau_\phi = 0$. Then $v_0 = v_1 = 0$ and, therefore,

$$f_1 = \frac{a}{2b - c},$$

$$f_0 = \frac{a}{2b - c} \left\{ 1 - \frac{2\delta b(b + c)n_1}{4b^2 - c^2} \right\}.$$

Period 1 expected prices will be equal to the one-shot game of complete information, while in period 0 they will be lower, provided we are in an equilibrium with positive n_1 .¹⁶

Example 2. Let τ_ϕ be arbitrarily large and $\tau_u = 0$. Then at any period, $n_t = 0$ and, approximately, $m_t = 0$, $v_t = 1/2g_t$, and in equilibrium, for $t = 1, 2$,

$$p_t^A = \frac{a}{2b - c - v_t} + \frac{q}{2b + c - v_t},$$

which implies that $f_t = (3/4)[a/(b - c)]$ and $g_t = (3/2)[1/(2b + c)]$.

Now the incentives on prices go in the opposite direction. Since higher prices signal higher q , firms have incentives to push prices upwards. It can be checked that in this unique equilibrium (with linear strategies), market shares are still a positive function of q , but this is irrelevant since in this limit case consumers do not need to look at market shares to learn about q .

¹⁶ If an equilibrium with $n_1 < 0$ exists, even with uninformative prices ($\tau_\phi = 0$), f_0 will be above $a/(2b - c)$.

Finally, let us focus on the effect of the informational content of market shares on pricing. If we let τ_u tend to zero, then n_1 tends also to zero. Thus, from the equilibrium equation for f_0 , this will be above the level of an equilibrium with $n_1 > 0$.¹⁷ In other words, observing past market shares increases the degree of competition, provided consumers infer higher relative quality from a higher market share.

In summary, when firms have private information on the relative quality differential q , prices potentially reveal some of this information. In this case, consumers generally update their beliefs by taking into account their private signals, current prices as well as past quantities. Firms try to influence consumers' beliefs by manipulating prices. However, incentives usually go in opposite directions depending on whether firms attempt to manipulate information revealed by prices or that revealed by quantities.¹⁸

Although it is difficult to have a complete characterization of the set of equilibria, it has been shown that, under a regularity condition, there exists an equilibrium where a high market share signals high quality. Hence, the main result of Section 4 is quite robust to variations on what firms know. With respect to the degree of competition, if we compare expected prices with those obtained in the game of complete information, then the answer is ambiguous, and this ambiguity largely depends on the relative informational content of prices and quantities. However, it is well known that the signalling role of prices tends to push prices upwards (see, for instance, Bagwell and Riordan (1991)). Thus, perhaps the only relevant issue is the effect on pricing of the observability of past market shares. If we take this view, the answer is analogous to that of Section 4: the informational role of market shares tends to make the market more competitive.

6. Concluding remarks

■ We have presented in this article an information-based theory of the value of market shares from the point of view of consumers and firms. To do so we have integrated a model of consumer learning with strategic decision making by firms. Consumers pay attention to past market shares because they provide an additional source of information about relative quality differences, given that they aggregate dispersed information of previous consumers. Firms compete for market shares because they attempt to manipulate consumers' learning. We have formalized these ideas in a very simple model, using a linear specification, describing the consumers' sources of information in a stylized fashion, and limiting the analysis to a two-period horizon.

Many issues are left open, mainly in connection with the induced price dynamics with an arbitrary horizon: Will consumers and firms eventually learn the realization of the quality differential? If so, how will consumers' learning pace affect the strategic incentives of firms and potential convergence to full-information pricing? Will the battle for market shares always be fiercer in the first stages of competition, or can price wars occur in intermediate stages of the product life-cycle? How will the pattern of price volatility be affected? We try to answer some of these questions in another article (Caminal and Vives, 1996).

¹⁷ This is true provided $2b + c - 3v_1 > 0$ in the equilibrium with $n_1 > 0$. This is checked in the proof of Proposition 2 in the Appendix.

¹⁸ In Milgrom and Roberts (1986) a relatively high price may signal high or low quality, depending on the difference between the marginal costs of producing a high- and a low-quality good. In our model, even though marginal costs are independent of quality, a high price may also signal high or low quality depending on the dominant signalling device: prices or market shares.

Appendix

■ Here we develop the model discussed in Section 5 and prove all the results.

□ **Updating beliefs.** Suppose consumers in period 0 conjecture that producer prices will take the posited linear form

$$\begin{aligned} p_0^A &= f_0 + g_0 q \\ p_0^B &= f_0 - g_0 q. \end{aligned} \quad (\text{A1})$$

Then,

$$\Delta p_0 = p_0^A - p_0^B = 2g_0 q \quad (\text{A2})$$

and $\Delta p_{i0} = p_{i0}^A - p_{i0}^B$ is a sufficient statistic of p_{i0}^A and p_{i0}^B in the estimation of q : $\Delta p_{i0} = 2g_0 q + \Delta \phi_{i0}$.

In period 0, consumers obtain information about q from the observation of their private signal and from the retail prices they have access to. Thus, Bayesian updating implies

$$E(q|I_{i0}) = m_0 s_{i0} + v_0 \Delta p_{i0},$$

where

$$m_0 = \frac{\tau_\epsilon}{\tau_{i0}}, \quad (\text{A3})$$

$$v_0 = \frac{g_0 \tau_\phi}{\tau_{i0}}, \quad (\text{A4})$$

and

$$\tau_{i0} = \tau_\epsilon + \tau_q + 2g_0^2 \tau_\phi. \quad (\text{A5})$$

According to our convention on the law of large numbers, consumers' aggregate beliefs, η_0 , will be given by

$$\eta_0 = (m_0 + 2g_0 v_0) q. \quad (\text{A6})$$

Notice that η_0 is an increasing function of q . As usual, Δx_0 is a sufficient statistic of x_0^A and x_0^B in the estimation of q :

$$\Delta x_0 = 2\eta_0 q - (b + c) \Delta p_0 + \Delta u_0 = 2zq + \Delta u_0, \quad (\text{A7})$$

where

$$z = [m_0 + g_0(2v_0 - (b + c))]. \quad (\text{A8})$$

Notice that z positive means that a high market share signals high quality. That is, when z is positive, $E(q|I_{i1})$ increases with Δx_0 .

Similarly, consumers in period 1 conjecture that producer prices will take the linear form

$$\begin{aligned} p_1^A &= f_1 + g_1 q + h_1 \Delta u_0 \\ p_1^B &= f_0 - g_0 q - h_0 \Delta u_0. \end{aligned} \quad (\text{A9})$$

Since, in principle, period 1 prices respond to Δu_0 , they are a more noisy signal of q than in period 0: $\Delta p_{i1} = 2(g_1 q + h_1 \Delta u_0) + \Delta \phi_{i1}$. Thus, consumer i 's beliefs in period 1 are given by

$$E(q|I_{i1}) = m_1 s_{i1} + v_1 \Delta p_{i1} + n_1 \Delta x_0,$$

where

$$m_1 = \frac{\tau_\epsilon}{\tau_{i1}}, \tag{A10}$$

$$v_1 = \frac{g_1 \tau_u \tau_\phi}{(4h_1^2 \tau_\phi + \tau_u) \tau_{i1}}, \tag{A11}$$

$$n_1 = \frac{\tau_u z}{\tau_{i1}}, \tag{A12}$$

and

$$\tau_{i1} = \tau_\epsilon + \tau_q + \frac{2g_1^2 \tau_u \tau_\phi}{4h_1^2 \tau_\phi + \tau_u} + 2\tau_u z^2. \tag{A13}$$

Finally, consumers' aggregate beliefs in period 1 are given by

$$\eta_1 = m_1 q + v_1 \Delta p_1 + n_1 \Delta x_0. \tag{A14}$$

□ **Equilibrium pricing.** In the last period, firm *A*'s optimization problem consists of choosing p_1^A in order to maximize

$$\Pi_1^A = (a + \eta_1 - bp_1^A + cp_1^B)p_1^A,$$

where η_1 is given by (A14). Notice that prices directly influence consumers' aggregate beliefs. Since firm *B* solves a similar optimization problem, equilibrium prices are given by

$$p_1^A = \frac{a}{2b - c - v_1} + \frac{\eta_1}{2b + c - v_1} \tag{A15}$$

$$p_1^B = \frac{a}{2b - c - v_1} - \frac{\eta_1}{2b + c - v_1},$$

while sales expected at the beginning of period 0 are

$$E(x_1^i | I_1) = b - (v_1)p_1^i. \tag{A16}$$

From equations (A2), (A7), (A8), and (A10) to (A15) we can now provide implicit expressions for the equilibrium values of f_1 , g_1 , and h_1 :

$$f_1 = \frac{a}{2b - c - v_1} \tag{A17}$$

$$g_1 = \frac{m_1 + 2g_1 v_1 + 2n_1(m_1 + 2g_0 v_0 - (b + c)g_0)}{2b + c - v_1} \tag{A18}$$

$$h_1 = \frac{2h_1 v_1 + n_1}{2b + c - v_1}. \tag{A19}$$

In period 0, firm *A* must choose p_0^A in order to maximize

$$\Pi_0^A = (a + \eta_0 - bp_0^A + cp_0^B)p_0^A + \delta E(p_1^A x_1^A | I_0),$$

where p_1^A and x_1^A are given by (A15) and (A16). Firm *B* solves a similar optimization problem. The first-order condition is

$$a + \eta_0 - (2b - v_0)p_0^A + cp_0^B + E\left\{ \delta(b - v_1)2p_1^A \frac{dp_1^A}{dp_0^A} \middle| I_0 \right\} = 0.$$

It is sufficient if the regularity condition in footnote 14 holds.

From (A14) and (A15),

$$\frac{dp_0^A}{dp_0^B} = -\frac{n_1(b+c)}{2b+c-3v_1}.$$

Using this expression, the first-order condition of firm *B*'s optimization problem, and the fact that

$$E(\eta_1|I_0) = (m_1 + 2v_1g_1 + 2zn_1)q,$$

we have

$$p_0^A = \frac{a}{2b-c-v_0} + \frac{\eta_0}{2b+c-v_0} - \frac{1}{2b-c-v_0} \frac{b+c}{2b+c-3v_1} \frac{2\delta(b-v_1)a}{2b-c-v_1} n_1 - \frac{2\delta(b-v_1)(b+c)n_1(m_1+2g_1v_1+2n_1z)}{(2b+c-v_0)(2b+c-v_1)(2b+c-3v_1)} q.$$

Finally, since η_0 is given by (A6), we can also provide implicit expressions for the equilibrium values of f_0 and g_0 :

$$f_0 = \frac{a}{2b-c-v_0} \left\{ 1 - \frac{2\delta(b-v_1)(b+c)n_1}{(2b+c-3v_1)(2b-c-v_1)} \right\} \tag{A20}$$

$$g_0(2b+c) = m_0 + 3g_0v_0 - \frac{2\delta(b-v_1)(b+c)n_1(m_1+2g_1v_1+2n_1z)}{(2b+c-v_1)(2b+c-3v_1)}. \tag{A21}$$

Thus, equilibrium is implicitly defined by equations (A3) to (A5), (A8), (A10) to (A13), and (A17) to (A21). We shall see that there may be multiple meaningful solutions to these equations.

Proof of Proposition 2. To check that $g_1 > 0$ is immediate. Equation (A12) indicates that v_1 and g_1 have the same sign. Equation (A12) implies that n_1z is always positive. Finally, from equation (A18), if v_1 is negative, then g_1 is positive.

Equilibria are characterized by the equations (A3) to (A5), (A8), (A10) to (A13), and (A17) to (A21). We shall show that there always exists a solution with $z > 0$. To do that we first rewrite the system.

Taking into account (A3) to (A5) and (A18), equation (A21) can be written as

$$g_0 = \frac{1}{2b+c} \left\{ \frac{\tau_\epsilon + 3g_0^2\tau_\phi}{\tau_\epsilon + \tau_q + 2g_0^2\tau_\phi} - \frac{2\delta(b-v_1)n_1g_1}{2b+c-3v_1} \right\}. \tag{A22}$$

If we plug this expression in the definition of z , equation (A8), we obtain

$$z = \frac{1}{2b+c} \left\{ \frac{b\tau_\epsilon + (b-c)g_0^2\tau_\phi}{\tau_\epsilon + \tau_q + 2g_0^2\tau_\phi} + \frac{2\delta(b-v_1)(b+c)^2n_1g_1}{2b+c-3v_1} \right\}. \tag{A23}$$

Next, using (A10) to (A13), we rewrite (A18):

$$g_1 = \frac{\tau_\epsilon + \frac{3g_1^2\tau_u\tau_\phi}{4h_1^2\tau_\phi + \tau_u} + 2\tau_u z^2}{(2b+c) \left(\tau_\epsilon + \tau_q + \frac{2g_1^2\tau_u\tau_\phi}{4h_1^2\tau_\phi + \tau_u} + 2\tau_u z^2 \right)}. \tag{A24}$$

From (A11), (A13), and A(18), we have

$$v_1 = \frac{(2b+c) \frac{g_1^2\tau_u\tau_\phi}{4h_1^2\tau_\phi + \tau_u}}{\tau_\epsilon + \frac{3g_1^2\tau_u\tau_\phi}{4h_1^2\tau_\phi + \tau_u} + 2\tau_u z^2}. \tag{A25}$$

Equation (A19) can also be written as

$$h_1 = \frac{n_1}{2b + c - 3v_1}. \tag{A26}$$

Equations (A22) to (A26) plus (A12) (after plugging in (A13)) constitute a system of six equations and six unknowns.

Let us now restrict the domain of these variables. Each variable can take values only in a closed and connected interval. For each variable we define the upper bound with an upper bar and the lower bound with a lower bar.

$$\begin{aligned} \underline{n}_1 &= 0 \text{ and } \bar{n}_1 = +\sqrt{\frac{\tau_u}{8(\tau_\epsilon + \tau_q)}}, \\ \underline{g}_1 &= \frac{\tau_\epsilon}{(2b + c)(\tau_\epsilon + \tau_q)} \quad \text{and} \quad \bar{g}_1 = \frac{1}{b + c}, \\ \underline{v}_1 &= 0 \quad \text{and} \quad \bar{v}_1 = \frac{(2b + c)\bar{g}_1\tau_\phi}{\tau_\epsilon + 3\bar{g}_1^2\tau_\phi}, \\ \underline{h}_1 &= 0 \quad \text{and} \quad \bar{h}_1 = \frac{\bar{n}_1}{2b + c - 3\bar{v}_1}, \\ \underline{g}_0 &= \frac{1}{2b + c} \left(\frac{\tau_\epsilon}{\tau_\epsilon + \tau_q} - \frac{2\delta b\bar{n}_1\bar{g}_1}{2b + c - 3\bar{v}_1} \right), \end{aligned}$$

and \bar{g}_0 is the unique positive solution to

$$\bar{g}_0 = \frac{\frac{b + c}{2b + c} \tau_\epsilon + 2\bar{g}_0^2 \tau_\phi}{(b + c)(\tau_\epsilon + \tau_q + 2\bar{g}_0^2 \tau_\phi)}.$$

Finally, depending on the values of the parameters, we define the interval for z differently:

$$\underline{z} = \frac{b}{2b + c} \frac{\tau_\epsilon}{\tau_\epsilon + \tau_q} \quad \text{if } (b - c)\tau_q > (b + c)\tau_\epsilon$$

and

$$\begin{aligned} &= \frac{b\tau_\epsilon + (b - c)2\bar{g}_0^2\tau_\phi}{(2b + c)(\tau_\epsilon + \tau_q + 2\bar{g}_0^2\tau_\phi)} \quad \text{otherwise.} \\ \bar{z} &= \frac{1}{2b + c} \left\{ \frac{b\tau_\epsilon + (b - c)\bar{g}_0^2\tau_\phi}{\tau_\epsilon + \tau_q + 2\bar{g}_0^2\tau_\phi} + \frac{2\delta b(b + c)^2\bar{n}_1\bar{g}_1}{2b + c - 3\bar{v}_1} \right\} \quad \text{if } (b - c)\tau_q > (b + c)\tau_\epsilon \end{aligned}$$

and

$$= \frac{1}{2b + c} \left\{ \frac{b\tau_\epsilon}{\tau_\epsilon + \tau_q} + \frac{2\delta b(b + c)^2\bar{n}_1\bar{g}_1}{2b + c - 3\bar{v}_1} \right\} \quad \text{otherwise.}$$

If we let A be the set of all values of $g_0, z, g_1, v_1, h_1,$ and n_1 in the intervals defined above, then A is a nonempty compact set. All bounds are finite, thus by construction A is compact. To check that A is nonempty is immediate. Perhaps some calculations are needed to illustrate the bounds of h_1 and g_0 . In the first case, notice that

$$2b + c - 3\bar{v}_1 = \frac{(2b + c)\tau_\epsilon}{\tau_\epsilon + 3\bar{g}_0^2\tau_\phi} > 0$$

and therefore \bar{h}_1 is strictly positive. Also,

$$\underline{g}_0 = \frac{1}{2b + c} \left(\frac{\tau_\epsilon}{\tau_\epsilon + \tau_q} - \frac{2\delta b\bar{n}_1\bar{g}_1}{2b + c - 3\bar{v}_1} \right) \leq \frac{1}{2b + c} \frac{\tau_\epsilon}{\tau_\epsilon + \tau_q} < \frac{\frac{b + c}{2b + c} \tau_\epsilon + 2\bar{g}_0^2 \tau_\phi}{(b + c)(\tau_\epsilon + \tau_q + 2\bar{g}_0^2 \tau_\phi)} = \bar{g}_0.$$

If we denote by Φ the mapping from \mathbb{R}^6 to \mathbb{R}^6 defined by (A22) to (A26) plus (A12), then clearly Φ is continuous in A .

The final step is to show that $\Phi(A)$ is contained in A , which in all cases involves simple calculations. Summarizing, Φ is a continuous function that maps A into A . Therefore there exists a fixed point with $z > 0$. From equation (A12) this implies that there always exists an equilibrium with $n_1 > 0$. *Q.E.D.*

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