Economics Letters 21 (1986) 113–116
North-Holland

RATIONING RULES AND BERTRAND–EDGEWORTH EQUILIBRIA IN LARGE MARKETS

Xavier VIVES

University of Pennsylvania, Philadelphia, PA 19104, USA

Received 7 February 1986

In a market with concave downward sloping demand and symmetric firms which compete in prices with constant marginal costs and capacity limits it is shown that the supports of the symmetric (mixed strategy) Nash equilibria converge to the unique competitive price provided that unsatisfied demand is allocated according to the surplus-maximizing rationing rule.

1. Introduction

Allen and Hellwig (A–H hereafter) (1984a) show for a market where firms compete in prices with capacity constraints that as the number of firms increases, Nash equilibria (in pure or mixed strategies) converge in distribution to a (perfectly) competitive price, but monopoly prices persist for any number of firms. In other words, the supports of the equilibrium mixed strategies do not necessarily converge to a subset of the competitive price set. We have, thus, that in Bertrand–Edgeworth mixed strategy equilibria, 'competition plays no role in the determination of the "highest" price in the market' [A–H (1984a, p. 16)].

A–H assume that rationing at the lowest price is made through a queueing system. Consumers in front of the line obtain their entire demand while others obtain nothing. We will refer to this way of allocating unsatisfied demand the *proportional* rationing scheme since once the lowest priced firms have satisfied a proportion α of the consumers there remains a proportion $1-\alpha$ of consumers to be served at higher prices. An alternative method is to have consumers buy first from the cheapest supplier and assume no income effects. In this case the high-priced portion of the demand curve is satisfied first. We will refer to this method as the *surplus-maximizing* rationing scheme. [1] We present below a model using the surplus-maximizing rationing scheme where the supports of the equilibrium mixed strategies do converge to the (unique) competitive price.

2. The model

Consider a symmetric n-firm oligopoly where each firm has constant marginal production costs. Inverse demand is given by a twice-continuously differentiable, strictly decreasing and concave

[1] The proportional rationing scheme has been used by Beckmann (1965) [see also Shubik (1959)], Dasgupta and Maskin (1986b), and Allen and Hellwig (1984,1985), the surplus-maximizing rationing scheme by Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchnik (1983), and Brock and Scheinkman (1985). Davidson and Deneckere (1982) discuss the implications of the proportional rationing scheme for the Kreps and Scheinkman (1983) result.

function $P(\cdot)$ on some bounded interval $[0, \bar{X}]$. Price is zero for outputs larger than or equal to $\bar{X}$. The demand function $D(\cdot)$ shares the properties of P in the appropriate range. Without loss of generality we assume marginal costs are zero. It is well known that under these conditions there is a unique and symmetric Cournot equilibrium in which each firm produces y_n, with $y_n < \bar{X}/n$, that total Cournot output ny_n is increasing with n and that the order of magnitude of the Cournot price is $1/n$ [see Szidarovsky and Yakowitz (1977) and Ruffin (1971), for example]. Suppose now that each firm has a capacity constraint k_n and that there is price competition with the surplus maximizing rationing rule. According to this rule the contingent demand for firm i given that l firms charge lower prices than p_i and that s firms charge p_i is $(D(p_i) - lk)/s$ provided this expression is positive and 0 otherwise. Proposition 1 below characterizes the Nash equilibria of the game.

Proposition 1. *Given n firms in the market and restricting attention to symmetric equilibria,*

(a) if $k_n \leqq y_n$ then each firm charges $P(nk)$,
(b) if $y_n < k_n < \bar{X}/(n-1)$ then there is a mixed strategy equilibrium where each firm randomizes according to the (atomless) distribution function ϕ_n with support $[\underline{p}_n, \bar{p}_n]$ obtaining expected profits $\bar{\pi}_n$.

$$\bar{p}_n = \arg\max\{p(D(p) - (n-1)k_n)\}, \qquad \bar{\pi}_n = \bar{p}_n(D(\bar{p}_n) - (n-1)k_n),$$

$$\underline{p}_n = \bar{\pi}_n/k_n \quad \textit{and} \quad \phi_n(p) = [(k_n - \bar{\pi}_n/p)/(nk_n - D(p))]^{1/(n-1)},$$

(c) if $k_n \geqq \bar{X}/(n-1)$ then each firm charges a zero price.

For example, if demand is linear, $p = a - X$, $y_n = a/(n+1)$, $\bar{X} = a$, $\bar{p}_n = (a - (n-1)k_n)/2$ and $\bar{\pi}_n = (\bar{p}_n)^2$ [see Brock and Scheinkman (1985)].

The proof of Proposition 1 is standard and follows along the lines of Levitan and Shubik (1972), and Kreps and Scheinkman (1983). A rough sketch of the argument is given. If (c) holds then $p_i = 0$ for all i is an equilibrium since if one firm charges a higher price it has no residual demand left. If firms were to charge some common positive price then any firm could increase its profit by undercutting the common price. If (a) holds then $p_i = P(nk_n)$ for all i is an equilibrium. If any firm undercuts $P(nk_n)$ it makes less since it still sells k_n but at a lower price. If a firm puts a higher price it has a residual demand $D(p) - (n-1)k_n$ but the optimal response is precisely to put $p = P(nk_n)$ since k_n is less than or equal to the Cournot output y_n; Cournot best response functions are decreasing (since demand is concave) and therefore the firm is effectively constrained by k_n. No other price can be an equilibrium. If (b) holds then no equilibrium in pure strategies exists and it can be shown, using arguments similar to Dasgupta and Maskin (1986) that there is an atomless mixed strategy equilibrium with support on a price interval, say $[\underline{p}_n, \bar{p}_n]$. The upper bound of the support must be the monopoly price on the residual demand left to a firm when all the rivals undercut its price, that is, $\bar{p}_n = \arg\max\{p(D(p) - (n-1)k_n)\}$. (Note that the firm will not be capacity constrained since $k_n > y_n$.) Now, given that the rivals use the strategy ϕ_n the expected profits to the firm of naming any price $p \in [\underline{p}_n, \bar{p}_n]$ must be the same. Denote them by $\bar{\pi}_n$. We have thus $\bar{\pi}_n = \bar{p}_n(D(\bar{p}_n) - (n-1)k_n)$. The expected profits when naming $\underline{p}_n$ are $\underline{p}_n k_n$, therefore $\underline{p}_n = \bar{\pi}_n/k_n$. Furthermore for any p in $[\underline{p}_n, \bar{p}_n]$, $\bar{\pi}_n = (1 - [\phi_n(p)]^{n-1})pk_n + [\phi_n(p)]^{n-1}p(D(p) - (n-1)k_n)$. Solving for $\phi_n(p)$ we get the desired expression. Note that $\underline{p}_n > P(nk_n)$ since by strict concavity of profit, $\bar{\pi}_n > P(nk_n)k_n$ and $\bar{\pi}_n = \underline{p}_n k_n$.

Following A–H we fix aggregate capacity to a level K, so that $k_n = K/n$, and let the number of firms go to infinity (A–H work with asymmetric cases as well while keeping individual capacities of the same order of magnitude). We have to consider three cases according to whether $K \lesseqgtr \bar{X}$. Recall that ny_n monotonically converges to $\bar{X}$. If $K < \bar{X}$ let $\bar{n} = \max\{n \in N\colon\ ny_n < K\}$ if the set is non-empty, otherwise let $\bar{n} = 1$. Then if $\bar{n} \geqq 2$ there is a mixed strategy equilibrium for $2 \leq n \leq \bar{n}$. Otherwise firms charge $p = P(K)$. If $K > \bar{X}$ let $\tilde{n} = \max\{n\colon\ 1 - 1/n < \bar{X}/K\}$. Then if $\tilde{n} \geqq 2$ for $2 \leqq n \leqq \tilde{n}$ firms randomize. Otherwise firms charge zero prices. If $K = \bar{X}$ then $y_n < k/n < \bar{X}/(n-1)$ for all n and we know that the supremum of the support of the mixed strategy ϕ_n, $\bar{p}_n$, solves in p, $pD'(p) + D(p) = (1 - 1/n)\bar{X}$. Since D is concave $pD'' + 2D'$ is negative and $\bar{p}_n$ monotonically converges to 0 at a rate $1/n$. [Just note that $\bar{p}_n \leqq \bar{X}/(n\,|\,D'(0)\,|)$]. Furthermore $\bar{\pi}_n = \bar{p}_n(D(\bar{p}_n) - (1 - 1/n)\bar{X})$ which using the first-order condition for $\bar{p}_n$ equals $(\bar{p}_n)^2\,|\,D'(\bar{p}_n)\,|$ and expected profits $\bar{\pi}_n$ decline monotonically to zero at a rate $1/n^2$. Obviously ($\underline{p}_n = n\bar{\pi}_n/\bar{X}$) the order of magnitude of $\underline{p}_n$ is $1/n$. For example, in the linear demand case $\bar{p}_n = a/2n$, $\underline{p}_n = a/4n$ and $\bar{\pi}_n = (a/2n)^2$. Proposition 2 summarizes the asymptotics of our example.

Proposition 2. *Fix K, then*

(i) *if $K < \bar{X}$ for n large enough all firms charge $P(K)$,*
(ii) *if $K = \bar{X}$ there is a symmetric mixed strategy equilibrium for all n with the supremum of its support converging monotonically to zero at a rate $1/n$. The order of magnitude of expected profits is $1/n^2$, and*
(iii) *if $K > \bar{X}$ for n large enough all firms charge a zero price.*

We see thus that in contrast to the A–H result the supports of our equilibrium strategies converge to the unique competitive price of the market. Under our assumptions on demand but using the proportional rationing scheme and with n firms in the market each with capacity k_n, $nk_n = K \leqq \bar{X}$, an equilibrium in pure strategies exists if and only if the competitive price is equal to the monopoly price p^m ($p^m = \arg\max\ p\ \min[K,\ D(p)]$) [A–H (1984a, p. 13)]. If this is not the case then there is a symmetric atomless mixed strategy equilibrium in which the supremum of the prices charged by a firm is the monopoly price p^m, which is independent of n [see A–H (1984a,b)]. As the number of firms grows the mixed strategy equilibria converge in distribution to the competitive price but the supports do not. The monopoly price is always named by a firm since given the proportional rationing scheme the residual demand left for firm i when it charges the highest price p_i is $s(p_{-i};\ k_n)D(p_i)$ where p_{-i} are the prices charged by the other firms and $s(p_{-i};\ k_n)$ is the proportion of customers left unserved by the rivals (each with capacity k_n). Now given that each rival uses the mixed strategy F_n and that firm i will be undercut almost surely the expected profits of firm i when charging a price p are given by $pD(p)$ $(\mathrm{E}s(p_{-i};\ k_n))$ where the expectation is taken according to the product of the $n-1$ distributions F_n. Therefore firm i names p^m 'independently of competition in the market'. On the other hand when the surplus-maximizing rationing rule is used the residual demand left to firm i when it is undercut by the rivals is $D(p_i) - (n-1)k_n$ and the firm names the monopoly price of the residual demand which now depends on the capacity of the rivals, $(n-1)k_n$. As the number of firms increases the highest price named by firm i, $\bar{p}_n$, necessarily goes to the competitive level since the quantity sold by the rivals $(n-1)K/n$ approaches the competitive quantity K. When the high priced portion of the demand schedule is served first, as it is the case with the surplus-maximizing rationing rule, there is more downward pressure on prices than when low priced firms serve a random sample of the consumers, as it is the case with the proportional rationing rule. In both cases nevertheless equilibrium price distributions converge weakly to competitive prices.

References

Allen, B. and M. Hellwig, 1984a, Bertrand–Edgeworth oligopoly in large markets, Discussion paper no. 135, Sonderforschungsbereich 21 (University of Bonn, Bonn). Forthcoming in Review of Economic Studies.

Allen, B. and M. Hellwig, 1984b, The range of equilibrium prices in Bertrand–Edgeworth duopoly, Discussion paper no. 141, Sonderforschungsbereich 21 (University of Bonn, Bonn).

Allen, B. and M. Hellwig, 1985, The approximation of competitive equilibria by Bertrand–Edgeworth equilibria in large markets, Discussion paper no. A-1, Sonderforschungsbereich 303 (University of Bonn, Bonn).

Beckmann, M.J., with the assistance of D. Hochstädter, 1965, Bertrand–Edgeworth duopoly revisited, in: Rudolf Henn, ed., Operations Research Verfahren, Vol. III (Hain, Meisenheim) 55–68.

Bertrand, J., 1883, Review of 'Théorie mathématique de la richesse sociale' and 'Recherches sur les principes mathématiques de la théorie des richesses', Journal des Savants, 499–508.

Dasgupta, P. and E. Maskin, 1986a, The existence of equilibrium in discontinuous economic games, 1: Theory, Review of Economic Studies 53.

Dasgupta, P. and E. Maskin, 1986b, The existence of equilibrium in discontinuous economic games, 2: Applications, Review of Economic Studies 53.

Davidson, C. and R. Deneckere, 1982, Long run competition in capacity, short run competition in price and the Cournot model, Mimeo.

Edgeworth, F.Y., 1925, The pure theory of monopoly, in: Edgeworth, papers relating to political economy, Vol. I, ch. E (Franklin, New York) 111–142.

Kreps, D.M. and J.A. Scheinkman, 1983, Quantity pre-commitment and Bertrand competition yield Cournot outcomes, Bell Journal of Economics 14, 326–338.

Levitan, R. and M. Shubik, 1972, Price duopoly and capacity constraints, International Economic Review 13, 111–122.

Osborne, M.J. and C. Pitchnik, 1983, Price competition in a capacity-constrained duopoly, Department of economics discussion paper series no. 185 (Columbia University, New York).

Ruffin, R., 1971, Cournot oligopoly and competitive behavior, Review of Economic Studies 38, 493–502.

Shubik, M., 1959, Strategy and market structure (Wiley, New York).

Szidarovsky, I. and S. Yakowitz, 1977, A new proof of the existence of uniqueness of the Cournot equilibrium, International Economic Review 18, 787–789.