

## **2. Aggregation of information in simple market mechanisms: how large is large?**

This chapter continues the study of simple market mechanisms in the context of markets with a finite number of participants. In Chapter 1 we have assumed that market participants were so small that they could not influence prices. What if this is not true and strategic behavior is possible? How large a market is needed for price-taking behavior to be a reasonable approximation? If a large market aggregates information, how fast does it happen? Is market power or lack of information aggregation the more important obstacle to efficiency? Does the answer to the questions depend on the type of market mechanism, say Cournot or auction? Does it depend on the type of uncertainty and information?

The theme of the chapter is to analyze the finite economy approximations to large markets, convergence to price-taking behavior and information aggregation, as well as its speed, as the economy grows large. In order to do so the chapter introduces the tools needed for the study of strategic behavior with incomplete information. There are at least two reasons to undertake the convergence analysis as a market grows large.

The first reason is to check consistency with the analysis in a continuum economy (as performed in Chapter 1). The equilibria in the continuum economy should be the limit of equilibria of finite economies and not an artifact of the continuum specification. For example, a method to analyze the convergence properties of the Cournot mechanism under incomplete information is to consider replica markets where demand is replicated as the number of firms increases, and obtain in this manner a well defined competitive limit market. With increasing marginal costs of production this can be understood as treating the size of the market (say the number of consumers) as an exogenous parameter and then making it grow. It can be seen then that the number of firms that would enter (and pay a fixed cost of entry) in a free entry equilibrium grows at the same rate as the market.

The second reason is practical and is to understand how large a market should be so that the results obtained in the large (continuum) market context are applicable to finite

economies, and to see how fast price-taking behavior, and information aggregation if this is the case, emerges as the economy grows large. Indeed, even if a large market aggregates information and first best efficiency is obtained in the limit, we may still wonder how large a market should be to approach efficiency. That is, if as the market grows large the full information competitive outcome is approached, how fast is this happening? What is the rate of convergence to efficiency? The issue is important; if convergence is obtained and yet its speed is very slow it means that a very large market is needed for the competitive approximation to work.

Here is a preview of the results we obtain in the chapter for the case of uncertainty of the independent values type, where as the market grows large a first best efficient outcome is obtained. We find that convergence is fast in auctions, in contrast to a Cournot market. Furthermore, we will see that the reason for the slow convergence in the Cournot market is the rate at which information is aggregated and not the rate at which market power is dissipated. In contrast, winner-takes-all markets like auctions tend to deliver aggregation of information more easily and faster, in the sense that with a smaller number of participants we are closer to the shared information outcome. As we have seen in Chapter 1, winner-takes-all markets force traders to condition effectively on more information when taking their decisions.

Section 1 presents a general  $n$ -firm Cournot model in the linear-normal setup. Section 2 analyzes convergence to price-taking behavior as the market grows large, providing in addition a foundation for the use of the continuum model. Section 3 studies convergence properties with costly information acquisition. Section 4 studies the case where convergence to the full information first best is obtained, and disentangles market power from information aggregation as different determinants of the convergence rate. Section 5 analyzes the convergence properties of the auction mechanism. The appendix collects some proofs.

## 2.1 A general linear-normal Cournot model

In this section we consider a linear-quadratic Cournot model in a homogenous product market with  $n$  firms, each having random production costs.<sup>1</sup> As in section 1.4 the information structure allows, as extremes, for private values (correlated types or cost parameters with perfect signals) and for a common value (with noisy signals). This model provides the finite economy counterpart of the linear-normal continuum models discussed Sections 1.2-1.4.

Consider thus a market for a homogenous product with  $m$  consumers, each with quasilinear preferences and having the net benefit function

$$U(x) - px \quad \text{with } U(x) = \alpha x - \beta x^2 / 2,$$

where  $\alpha$  and  $\beta$  are positive parameters and  $x$  the consumption level. Suppose that there are  $n$  firms in the market with firm  $i$  producing  $x_i$  and a total output of  $\sum_{i=1}^n x_i$ . This gives rise to the inverse demand  $P_m\left(\sum_{i=1}^n x_i\right) = \alpha - \beta x_m$  where  $x_m = \left(\sum_{i=1}^n x_i\right) / m$ . The size of the market is parameterized by the number of consumers  $m$ . We will consider here an  $n$ -replica market where  $n = m$  and  $x_n$  is the average or per capita output. Firm  $i$  produces according to a quadratic cost function

$$C(x_i, \theta_i) = \theta_i x_i + \frac{\lambda}{2} x_i^2$$

Where  $\theta_i$  is a random parameter and  $\lambda > 0$ . Total surplus is therefore given by

$$TS = nU(x_n) - \sum_{i=1}^n C(x_i, \theta_i) \quad \text{and} \quad \text{per capita surplus}$$

$$\text{by } TS/n = U(x_n) - \left(\sum_{i=1}^n C(x_i, \theta_i)\right) / n.$$

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<sup>1</sup> Sections 2.1 and 2.2 are based on Vives (2007).

As we will see below this replica market can also be interpreted as a market parameterized by the number of consumers and where firms can enter freely paying a positive fixed entry cost. Then the free entry number of firms is of the order of the number of consumers. A large market then is a market with a large number of consumers.

The information structure is the finite number of firms version of the one considered in Section 1.4 (see Section 2.3 in the Technical Appendix). Ex-ante, before uncertainty is realized, all firms face the same prospects. The vector of random variables  $(\theta_1, \dots, \theta_n)$  is jointly normally distributed with  $E[\theta_i] = \bar{\theta}$ ,  $\text{var}[\theta_i] = \sigma_\theta^2$ , and  $\text{cov}[\theta_i, \theta_j] = \zeta \sigma_\theta^2$ , for  $j \neq i$ ,  $0 \leq \zeta \leq 1$ . It follows that the average parameter  $\tilde{\theta}_n \equiv \left(\sum_{i=1}^n \theta_i\right)/n$  is normally distributed with mean  $\bar{\theta}$ ,  $\text{var}[\tilde{\theta}_n] = (1 + (n-1)\zeta)\sigma_\theta^2/n$ , and  $\text{cov}[\tilde{\theta}_n, \theta_i] = \text{var}[\tilde{\theta}_n]$ .

Firm  $i$  receives a signal  $s_i = \theta_i + \varepsilon_i$ , where  $\varepsilon_i \square N(0, \sigma_\varepsilon^2)$ ,  $\text{cov}[\varepsilon_i, \varepsilon_j] = 0$  for  $j \neq i$ , and  $\text{cov}[\theta_i, \varepsilon_j] = 0$  for all  $i$  and  $j$ . Under the normality assumption (as well as with the generalized affine information structure) conditional expectations are affine. We let as before  $\xi \equiv \tau_\varepsilon / (\tau_\theta + \tau_\varepsilon)$ .

There is common knowledge about payoff functions and distributions of random variables. The only (potential) private information of firm  $i$  is its private signal  $s_i$ .

Remark: This model can also be interpreted as the finite economy counterpart of the continuum model of Section 1.4. Indeed, the profits of firm  $i$  can be rewritten as

$$\pi_i = \left( \alpha - \theta_i - \left( \frac{\beta}{n} + \frac{\lambda}{2} \right) x_i - \frac{\beta}{n} \sum_{j \neq i} x_j \right) x_i = \left( \alpha - \theta_i - \frac{\lambda}{2} x_i - \frac{\beta}{n} \sum_{j=1}^n x_j \right) x_i,$$

which in the limit as  $n$  tends to infinity yields

$$\pi_i = (\alpha - \theta_i - (\lambda/2)x_i - \beta x)x_i$$

where  $x$  is the average output. In this interpretation with Cournot or Bertrand competition with product differentiation (with  $\omega_1 = \lambda/2$  and  $\omega_2 = \beta$  in Section 1.4) the limit market is monopolistically competitive.

### Equilibrium

The following proposition characterizes the Bayesian Cournot equilibrium and the price-taking Bayesian Cournot equilibrium (denoted by a superscript  $c$  for “competitive”). The proof is standard (but a good exercise for the reader) and is presented in the Appendix of the chapter.

Proposition 2.1. There is a unique Bayesian Cournot equilibrium and a unique price-taking Bayesian Cournot equilibrium. They are symmetric, and affine in the signals. The strategies of the firms are given (respectively) by:

$$X_n(s_i) = b_n(\alpha - \bar{\theta}) - a_n(s_i - \bar{\theta}) \quad , \quad \text{where} \quad a_n = \frac{\xi}{(2\beta/n) + \lambda + \beta\zeta\xi((n-1)/n)} \quad , \quad \text{and}$$

$$b_n = \frac{1}{\lambda + \beta((1+n)/n)} ;$$

$$X_n^c(s_i) = b_n^c(\alpha - \bar{\theta}) - a_n^c(s_i - \bar{\theta}) \quad , \quad \text{where} \quad a_n^c = \frac{\xi}{\beta/n + \lambda + \beta\zeta\xi((n-1)/n)} \quad , \quad \text{and} \quad b_n^c = \frac{1}{\lambda + \beta} .$$

Remark: From the F.O.C. of profit maximization it is immediate that in equilibrium expected profits for firm  $i$  are given by  $E[\pi_n] = \left(\frac{\lambda}{2} + \frac{\beta}{n}\right) E\left[(X_n(s_i))^2\right]$ .

Remark: In the case of independent values (i.e.  $\zeta = 0$  and  $\tau_\varepsilon = \infty$ ) the formulae are valid for a general distribution of the uncertainty.

Remark: The replica market considered can, in fact, be the outcome of free entry in a market parameterized by its size. Consider a market with  $m$  consumers (the size of the market),  $n$  firms producing a total output of  $\sum_{i=1}^n x_i$  and inverse demand  $P_m\left(\sum_{i=1}^n x_i\right) = \alpha - \beta\left(\sum_{i=1}^n x_i\right)/m$ . Suppose now that at a first stage firms decide whether to enter the market or not. If a firm decides to enter it pays a fixed cost  $F > 0$ . At a second stage each active firm  $i$ , upon observing its signal  $s_i$ , sets an output level. Given that  $n$  firms have entered, a Bayesian Cournot equilibrium is realized. Suppose that parameters are in a range that for any cost realization a firm wants to produce. Given our assumptions, for any  $n$  there is a unique, and symmetric, equilibrium yielding expected profits  $E[\pi_n]$  for each firm (similarly as in Proposition 2.1). A free entry equilibrium is a subgame-perfect equilibrium of the two-stage game. A subgame-perfect equilibrium requires that for any entry decisions at the first stage, a Bayesian-Nash equilibrium in outputs obtains at the second stage. Given a market of size  $m$ , the free entry number of firms  $n^*(m)$  is approximated by the solution to  $E[\pi_n] = F$  (provided  $F$  is not so large to prevent any entry). It can be checked that  $n^*(m)$  is of the same order as  $m$  (Vives (2002)). This means that the ratio of consumers to firms is bounded away from zero and infinity for any market size. We can reinterpret, therefore, the replica market as a free entry market parameterized by market size.

### Measures of speed of convergence

We examine next the convergence properties of this finite Cournot market as the economy grows. Before stating the convergence results we will recall measures of speed of convergence. (See Section 3.2 in the Technical Appendix for a fuller development and related definitions and measures.) Whenever  $n^{-\upsilon} b_n \xrightarrow{n} k$  for some nonzero constant  $k$  the sequence (of real numbers)  $b_n$  is of the *order*  $n^\upsilon$ , with  $\upsilon$  a real number. We say that the sequence of random variables  $\{y_n\}$  converges in *mean square* to zero at the rate  $1/\sqrt{n^\Gamma}$  (or that  $y_n$  is of the order  $1/\sqrt{n^\Gamma}$ ) if  $E[(y_n)^2]$  converges to zero at the rate

$1/n^r$  (i.e.  $E[(y_n)^2]$  is of the order  $1/n^r$ ). Given that  $E[(y_n)^2] = (E[y_n])^2 + \text{var}[y_n]$ , a sequence  $\{y_n\}$  such that  $E[y_n] = 0$  and  $\text{var}[y_n]$  is of the order of  $1/n$ , converges to zero at the rate  $1/\sqrt{n}$ . For example, if the random parameters  $(\theta_1, \dots, \theta_n)$  are i.i.d. with finite variance, then  $\tilde{\theta}_n - \bar{\theta}$  converges (in mean square) to 0 at the rate of  $1/\sqrt{n}$  because  $E[\tilde{\theta}_n - \bar{\theta}] = 0$  and  $\text{var}[\tilde{\theta}_n] = \sigma_\theta^2/n$ .

A more refined measure of convergence speed for a given convergence rate is provided by the *asymptotic variance*. Suppose that  $\{y_n\}$  is such that  $E[y_n] = 0$  and  $E[(y_n)^2] = \text{var}[y_n]$  converges to 0 at the rate  $1/n^r$  for some  $r > 0$ . Then the asymptotic variance is given by the constant  $AV = \lim_{n \rightarrow \infty} n^r \text{var}[y_n]$ . A higher asymptotic variance means that the speed of convergence is slower. It is worth noting that if the sequence  $\{y_n\}$  is normally distributed then  $\sqrt{n^r}(y_n)$  converges in distribution to  $N(0, AV)$ . Indeed, a normal random variable is characterized by mean and variance and we have that  $\text{var}[\sqrt{n^r}(y_n)] = n^r \text{var}[y_n]$  tends to  $AV$  as  $n$  tends to infinity.

## 2.2 Convergence to price taking in a Cournot market

We show in this section that Bayesian Cournot equilibria converge to (Bayesian) price-taking equilibria as  $n$  tends to infinity. This justifies the use of the continuum model as an approximation to the large Cournot market. We characterize also the speed at which this convergence occurs.

We consider, in turn, convergence to price taking and convergence to the continuum model as the economy is replicated. The following proposition characterizes the convergence of the Bayesian Cournot equilibrium to a price-taking equilibrium. The proof is in the Appendix to the chapter.

Proposition 2.2. As the market grows large the market price  $p_n$  (at the Bayesian Cournot equilibrium) converges in mean square to the price-taking Bayesian Cournot price  $p_n^c$  at the rate of  $1/n$ . (That is,  $E\left[(p_n - p_n^c)^2\right]$  tends to 0 at the rate of  $1/n^2$ .) The difference between (per capita) expected total surplus at the market outcome and at the price-taking Bayesian Cournot equilibrium  $(ETS_n^c - ETS_n)/n$  is of the order of  $1/n^2$ .

As the market grows large market power (in terms of the margin over marginal cost) dissipates at the rate of  $1/n$  and the welfare loss with respect to the price-taking equilibrium at the rate of  $1/n^2$ . As we will see in Section 2.3.2 these are the same rates of convergence as in the Cournot oligopoly with no uncertainty. Price-taking Bayesian Cournot equilibria provide the right benchmark of comparison because they maximize ETS subject to the use of decentralized strategies (Proposition 1.2).

The following proposition characterizes convergence of the Bayesian Cournot equilibria to the price-taking equilibrium of the continuum economy as the market grows large. (The proof is in the Appendix to the chapter.)

Proposition 2.3. As the economy is replicated the Bayesian Cournot equilibrium converges to the equilibrium in the continuum limit economy:  $X(s_i) = b(\alpha - \bar{\theta}) - a(s_i - \bar{\theta})$ , where  $a = \xi / (\lambda + \beta\zeta\xi)$  and  $b = 1 / (\lambda + \beta)$ . The Bayesian Cournot price  $p_n$  converges (in mean square) to  $p = \alpha - \beta(b(\alpha - \bar{\theta}) - a(\bar{\theta} - \bar{\theta}))$  at the rate of  $1/\sqrt{n}$  and  $\sqrt{n}(p_n - p)$  converges in distribution to  $N(0, \beta^2 a^2 ((1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2))$ .

Convergence to the equilibrium of the continuum economy happens at the rate  $1/\sqrt{n}$  at which the average error in the signals of the agents  $\frac{1}{n} \sum_{i=1}^n \varepsilon_i$  tends to zero. Convergence is slower, according to the asymptotic variance  $\beta^2 a^2 ((1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2)$ , with larger  $\sigma_\theta^2$  or  $\beta$ ,

and faster with larger  $\varsigma$  or  $\lambda$ . A larger  $\rho$  means that we are closer to a common value environment. The effect of an increase in  $\sigma_\varepsilon^2$  is ambiguous: the direct effect is to slow down convergence but the indirect effect is to lower the response to information, which has the opposite effect.

In summary, in the context of a general Cournot model, that allows for private or common values, we have checked that the equilibria obtained in the continuum Cournot markets in Chapter 1 are not an artifact but the limit of equilibria in finite economies. Furthermore, the convergence rate to the limit equilibrium is  $1/\sqrt{n}$  for prices, where  $n$  is the “size” of the market, and convergence is slower for higher prior uncertainty and faster when closer to a common value environment. Finally, convergence to price-taking is faster, at the rate of  $1/n$  for prices and  $1/n^2$  for the welfare loss (i. e. the deadweight loss with respect to price-taking).

### 2.3 Endogenous information acquisition

The convergence result can be extended to the case of costly information acquisition. Consider the finite economy counterpart of the continuum model in Section 1.6. That is, a sequence of replica markets with the  $n$ -replica consisting of  $n$  symmetric firms (each with cost  $C(x_i) = \lambda x_i^2 / 2$   $\lambda \geq 0$ ) and  $n$  consumers giving rise to an inverse demand  $p = \theta - \beta \left( \sum_{i=1}^n x_i \right) / n$ , where  $\sum_{i=1}^n x_i$  is the total output in the market. This model is a slight variation of the one presented in Section 2.1 with a common value information structure.

Competition proceeds in two stages. At the first stage firm  $i$  contracts the precision of its signal  $\tau_{\varepsilon_i}$  at constant per unit cost  $c \geq 0$  for a total cost  $c\tau_{\varepsilon_i}$ . For convenience and given that  $\xi_i \equiv \tau_{\varepsilon_i} / (\tau_\theta + \tau_{\varepsilon_i})$  we will consider the purchase of “precision”  $\xi_i$ , for a given  $\tau_\theta = \left( \sigma_\theta^2 \right)^{-1}$  at cost for firm  $i$

$$\varphi(\xi_i) = \frac{c}{\sigma_0^2} \frac{\xi_i}{1 - \xi_i}.$$

The purchased precisions are observable and at the second stage firms compete in quantities once each firm has received its private signal. Let us consider in turn the cases of decreasing returns ( $\lambda > 0$ ), constant returns ( $\lambda = 0$ ), and discuss the effects of the observability of information precision purchases.

### 2.3.1 Decreasing returns

Consider the second stage where firm  $i$  has a signal of precision  $\xi_i$ . It is possible to show, similarly to the proof of Proposition 2.1, that there is a unique Bayesian-Cournot equilibrium and it is given by

$$X_{in}(s_i) = a_{in}(s_i - \bar{\theta}) + b_n \bar{\theta} \quad \text{for } i = 1, \dots, n,$$

where

$$a_{in} = \frac{n}{\beta} \left( \frac{\gamma_{in}}{1 + \sum_{j=1}^n \gamma_{jn}} \right), \quad b_n = \frac{\lambda + \beta/n}{(\lambda + \beta)(\lambda + 2\beta/n)} \quad \text{and} \quad \gamma_{in} = \frac{\xi_i \beta/n}{(\lambda + 2\beta/n) - \xi_i \beta/n}.$$

The expected profits of firm  $i$  are given by  $E[\pi_{in}] = \left( \frac{\lambda}{2} + \frac{\beta}{n} \right) E[(X_{in}(s_i))^2]$  and the first stage payoff for given precisions is  $\Pi_{in}(\xi_1, \dots, \xi_n) = E[\pi_{in}] - \varphi(\xi_i)$ . Suppose that the purchases of information precision are observable. A Nash equilibrium of the game with payoffs  $\Pi_{in}(\xi_1, \dots, \xi_n)$  will constitute a subgame-perfect equilibrium of the two-stage game. Indeed, a subgame-perfect equilibrium requires that for any profile of precision purchases at the first stage, a Bayesian-Nash equilibrium in outputs obtains at the second stage. It can be checked that  $\Pi_{in}$  is strictly quasiconcave in  $\xi_i$  (i.e.  $\frac{\partial^2 \Pi_{in}}{\partial \xi_i^2} < 0$  for

$\frac{\partial \Pi_{in}}{\partial \xi_i} = 0$ ) and that competition is of the strategic substitutes type ( $\frac{\partial^2 \Pi_{in}}{\partial \xi_i \partial \xi_j} < 0$  for  $j \neq i$ ). It

follows then that there is a unique symmetric equilibrium  $\xi_{in}^* = \xi_n^*$ ,  $i = 1, \dots, n$ .<sup>2</sup> Let

$\phi_n(\xi) \equiv \frac{\partial \Pi_{in}}{\partial \xi_i} \Big|_{\substack{\xi_j = \xi, \\ j=1, \dots, n}}$ . The unique symmetric subgame-perfect equilibrium of the two-stage

game  $\xi_n^*$  solves in  $\xi$ ,  $\phi_n(\xi) = 0$  provided the solution is positive and equals zero otherwise. Some computations show that  $\gamma_n$

$$\phi_n(\xi) = \frac{(\lambda + 2\beta/n)(1 + (n-1)\gamma_n) + (n-1)\gamma_n \xi \beta/n}{((\lambda + 2\beta/n)(1 + (n-1)\gamma_n) - (n-1)\gamma_n \xi \beta/n)^3} \left(\frac{\lambda}{2} + \frac{\beta}{n}\right) \left(\frac{\beta}{n}\right)^4 \sigma_\theta^2 - \frac{c}{\sigma_\theta^2} \frac{1}{(1-\xi)^2},$$

where  $\gamma_n = \frac{\beta \xi/n}{(\lambda + 2\beta/n) - \beta \xi/n}$ . The solution  $\xi_n^*$  will be positive if  $\sigma_\theta^2 > 2\sqrt{c\left(\frac{\lambda}{2} + \frac{\beta}{n}\right)}$ . It is

easily checked that  $\xi_n^*$  tends to

$$\xi^* = \max \left\{ 0, \frac{\sigma_\theta^2 - \sqrt{2c\lambda}}{\beta\sqrt{2c/\lambda} + \sigma_\theta^2} \right\}$$

as  $n$  tends to infinity. This is exactly the solution obtained in the continuum economy in Section 1.6 when  $\lambda > 0$ .

In order to perform the welfare analysis, consider the problem of a planner that has to purchase an aggregate signal, with the same cost structure as above and that sets output to maximize expected total surplus. At the first best solution the result is that as  $n$  tends to infinity the first best optimal expenditure on information converges to zero in per capita terms and the precision of the aggregate signal goes to infinity. The argument is as follows. Consider the  $n$ -replica market and suppose there is a center which purchases an aggregate signal  $\tilde{s}_n$  with precision  $1/w_n$ . Given the pooled signal  $\tilde{s}_n = \left(\sum_{i=1}^n s_i\right)/n$ , expected per capita total surplus (gross of information costs), conditional on  $\tilde{s}_n$ , is  $E[\theta | \tilde{s}_n]x - (\beta + \lambda)x^2/2$  where  $x$  is average production. Optimal production is given by

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<sup>2</sup> See Vives (1988), and Section 2.3.2 and Section 5 in Vives (1999).

$X^o(\tilde{s}_n) = E[\theta | \tilde{s}_n] / (\beta + \lambda)$ . Expected total surplus as a function of the precision of the signal  $1/w_n$  is then

$$\frac{n}{2(\beta + \lambda)} E\left[\left(E[\theta | \tilde{s}_n]\right)^2\right] - \frac{c}{w_n} = \frac{n}{2(\beta + \lambda)} \left( \frac{\sigma_0^2}{\sigma_0^2 + w_n} \sigma_\theta^2 + \bar{\theta}^2 \right) - \frac{c}{w_n}.$$

It follows then that the optimal purchase of information precision is

$$\frac{1}{w_n^o} = \max \left\{ \left( \frac{\sqrt{n}}{\sqrt{2(\beta + \lambda)c}} - \frac{1}{\sigma_0^2} \right), 0 \right\}.$$

which tends to infinity as  $n$  goes to infinity. Nevertheless, the expenditure on information in per capita terms converges to zero as  $n$  tends to infinity since  $(c/nw_n^o) \xrightarrow{n \rightarrow \infty} 0$ . In the limit, per capita expenditure on information is zero and the full information first best is achieved.

### 2.3.2 Constant returns

With constant returns to scale firms will purchase information ( $\xi_n^* > 0$ ) if  $n$  is large enough;  $\xi_n^*$  will be decreasing in  $n$  for large  $n$  and will eventually converge to 0.<sup>3</sup> Recall that no equilibrium exists with costly research in the continuum economy (Section 1.6). The key observation is that with a finite number of firms expected profits are positive at the market equilibrium and incentives to acquire information are not destroyed. With a large number of firms, given that a firm's rivals are purchasing some information, the optimal response of the firm is also to acquire some information. With many firms and constant returns, all firms purchase a vanishing amount of information. The average precision in the market converges to zero as  $n$  goes to infinity. No equilibrium exists with costly information acquisition in the continuum economy but zero average expenditure

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<sup>3</sup> This follows from  $\sigma_0^2 > 2\sqrt{c\beta/n}$  for  $n$  large enough and from the expression of  $\phi_n(\xi)$  when  $\lambda = 0$ .

on information ( $\xi^* = 0$ ) is the limit of a unique sequence of subgame-perfect equilibria of the finite approximating economies.

With constant returns to scale the analysis of market optimality is more delicate. We have found that in the continuum limit no equilibrium exists but for any finite  $n$  the equilibrium does exist and involves a vanishing but positive average expenditure on information. What is the limit of per capita total expected surplus as the market grows? It can be checked (Exercise 2.1) that the total precision of information in the market tends to infinity as we replicate the economy (it is of the order of magnitude  $\sqrt{n}$ ) while the average expenditure on research goes to zero (it is of the order  $1/\sqrt{n}$ ). This implies that in the limit first best efficiency is obtained! The explanation is as follows. Fix a large  $n$ . We know then that every firm buys a small amount of information. If firms were price takers almost an efficient outcome would obtain: with constant returns (and the affine information structure) the market aggregates information and average expenditure on information is small. By increasing  $n$ , firms eventually become price takers; they always buy some information, and the average expenditure on research tends to zero. The limit is thus the first best outcome.

Li, McKelvey, and Page (1987) consider also information acquisition with constant returns to scale technology but they do not replicate demand when the number of firms increases. They find that the aggregate purchase of information may be positive in the competitive limit even though the purchase of an individual firm is zero. The welfare analysis indicates that the total expenditure on information acquisition undertaken by the market can be either too much, if the cost of research is low, or too little, if the cost of research is high. It is worth noting that, when examining the convergence properties of equilibria without replicating demand, the “limit” market is not well defined. Consider the Cournot model without uncertainty and with constant marginal costs: if demand is not replicated when increasing the number of firms, individual outputs are zero in the limit while aggregate output equals the competitive output.

### 2.3.3 Strategic aspects

In the continuum model of Section 1.6 whether information precision purchases were observable or not did not matter. In a finite economy with  $n$  firms it is no longer true that secret and overt information acquisition have the same consequences. Indeed, if precisions are observable there is a strategic effect with a finite number of players. It is possible to show that equilibrium precisions will be larger in the two-stage game with observable precisions. This is true, in fact, irrespective of whether market competition is of the strategic substitutes or complements variety. The reason is that in both cases there is an incentive to overinvest strategically in information acquisition. By acquiring more information in the Cournot (Bertrand) case the rivals respond less (more) to their information. This makes the residual demand faced by the firm more variable, and it is good for expected profits provided that the firm has some information.

The consequences for equilibrium expected profits (including costs of information acquisition) are different though in Bertrand and Cournot. Consider firms with symmetric cost functions. Then with price competition expected profits increase in the strategic two-stage game -in relation to the case where information acquisition is secret- because a firm likes the rival to be better informed. Competition at acquiring information is then of the strategic complements variety. With quantity rivalry, competition at acquiring information is of the strategic substitutes type. It turns out that equilibrium expected profits decrease in the strategic two-stage game (Hauk and Hurkens (2001)). This is not immediate because all firms have acquired more information.<sup>4</sup>

As the number of firms grows the outcome of any of the two games (one shot or simultaneous move and two-stage) in the Cournot market converges to the outcome we have obtained in the continuum economy. The reason is that, as it should be expected, the strategic effect disappears.

#### 2.3.4 Summary

With costly information acquisition in the Cournot market:

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<sup>4</sup> See Vives (Ch.8, 1999) for a full analysis of the incentives for firms to acquire and share information in oligopoly.

- The equilibria of the limit continuum economy can be rationalized as the limit of equilibria of finite economies provided returns are decreasing.
- With constant returns the limit economy is not well-behaved and there is no equilibrium despite the fact that finite economies are close to efficiency when large.
- When there is a finite number of firms and information acquisition is observable firms tend to overinvest strategically (and the same would be true if competition were to be à la Bertrand in a product differentiation environment).

#### 2.4 Convergence to the first best: market power and information aggregation

We study next the case in which the limit is first best efficient (with i.i.d. uncertainty) and the market aggregates information, and we decompose the speed of convergence in terms of information aggregation and market power effects.

##### 2.4.1 An independent values Cournot model<sup>5</sup>

Assume now in the model of Section 2.1 that the random cost parameters ( $\theta_i$ ) are drawn independently from a distribution with bounded support (independent values model). In this case convergence to the first best will be obtained. In order to disentangle market power and information aggregation effects we consider also the two full information regimes (Cournot and competitive) on top of the private information Bayesian equilibria. Variables in the non-strategic regimes are denoted by the superscript c (“competitive”); variables in the full information regimes are denoted by the superscript f (“full information”). Table 2.1 presents the different regimes and the corresponding notation for strategies.

At private information equilibria the strategy of firm  $i$  depends only on its type  $\theta_i$ . At full information equilibria it will depend on the realization of all types, which given the structure of the model can be summarized in  $(\theta_i, \tilde{\theta}_n)$ , where  $\tilde{\theta}_n = \frac{1}{n} \sum_{j=1}^n \theta_j$ . A unique

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<sup>5</sup> This section follows Vives (2002).

(and linear) equilibrium for each regime is easily derived (similar to the proof of Proposition 2.1).

**Table 2.1**

Information \	Strategic	Price-Taking
Private	Bayesian Cournot $X_n(\theta_i)$	Price-taking Bayesian Cournot $X_n^c(\theta_i)$
Full	Cournot $X_n^f(\theta_i, \tilde{\theta}_n)$	Competitive $X_n^{fc}(\theta_i, \tilde{\theta}_n)$

**Table 2.1**  
Regimes and strategies

Assuming that the underlying parameters are such that for any cost realization each firm wants to produce a positive quantity in any of the regimes<sup>6</sup> and letting  $\beta = 1$  we find the following:

$$X_n(\theta_i) = b_n(\alpha - \bar{\theta}) - a_n(\theta_i - \bar{\theta}); \quad a_n = (\lambda + 2/n)^{-1}, \quad b_n = (\lambda + (1+n)/n)^{-1}$$

$$X_n^c(\theta_i) = b_n^c(\alpha - \bar{\theta}) - a_n^c(\theta_i - \bar{\theta}); \quad a_n^c = (\lambda + 1/n)^{-1}, \quad b_n^c = (\lambda + 1)^{-1}$$

$$X_n^f(\theta_i, \tilde{\theta}_n) = b_n^f(\alpha - \bar{\theta}) - a_n^f(\theta_i - \tilde{\theta}_n); \quad a_n^f = a_n^c, \quad b_n^f = b_n$$

$$X_n^{fc}(\theta_i, \tilde{\theta}_n) = b_n^{fc}(\alpha - \tilde{\theta}_n) - a_n^{fc}(\theta_i - \tilde{\theta}_n); \quad a_n^{fc} = \lambda^{-1}, \quad b_n^{fc} = b_n^c.$$

---

<sup>6</sup> This means that the relative inefficiency of firms is never so large as to imply the shutdown of inefficient producers. A small  $\lambda$  constraints the support of  $\theta_i$  in order for outputs to be positive.

Average outputs in each case are easily obtained:  $x_n^c = b_n^c(\alpha - \bar{\theta}) - a_n^c(\tilde{\theta}_n - \bar{\theta})$ ,  $x_n = b_n(\alpha - \bar{\theta}) - a_n(\tilde{\theta}_n - \bar{\theta})$ ,  $x_n^f = b_n(\alpha - \tilde{\theta}_n)$  and  $x_n^{fc} = b_n^c(\alpha - \tilde{\theta}_n)$ . In the price-taking markets (i.e. when there is no market power difference) expected outputs (and prices) are the same with full and private information. Market power lowers expected output from  $b_n^c(\alpha - \bar{\theta})$  in the price-taking regimes to  $b_n(\alpha - \bar{\theta})$  in the regimes with market power ( $b_n < b_n^c$ ). Furthermore, the Bayesian Cournot equilibrium outputs are the less sensitive and the full information price-taking outputs are the most sensitive to own costs; both intermediate regimes have an equal intermediate sensitivity ( $a_n < a_n^f = a_n^c < a_n^{fc}$ ). Observe that keeping market power constant a firm reacts more to its cost realization with full information.

#### 2.4.2 Convergence rates

The following proposition characterizes the convergence of the Bayesian Cournot equilibrium to the full information competitive equilibrium as the market grows large and decomposes its rate into market power and information aggregation effects. The proof is similar to the proof of Proposition 2.2 and is left as an exercise.

Proposition 2.4. As the market grows large the Bayesian Cournot equilibrium price  $p_n$  converges (in mean square) to the full information competitive price  $p_n^{fc}$  at the rate of  $1/\sqrt{n}$ . The (per capita) expected deadweight loss at the market outcome  $(ETS_n^{fc} - ETS_n)/n$  is of the order of  $1/n$ . The orders of magnitude are driven by information aggregation and not by market power:

(i) Information is aggregated at the rate  $1/\sqrt{n}$  while market power is dissipated faster, at the rate  $1/n$ :  $P_n^c - p_n^{fc}$  and  $p_n^c - p_n^f$  are both of the order of  $1/\sqrt{n}$ , and  $p_n^f - p_n^{fc}$  and  $P_n - P_n^c$  are both of the order of  $1/n$ .

(ii)  $(ETS_n^{fc} - ETS_n^c)/n$  and  $(ETS_n^f - ETS_n)/n$  are of the order of  $1/n$ , while  $(ETS_n^c - ETS_n)/n$  and  $(ETS_n^{fc} - ETS_n^f)/n$  are of the order of  $1/n^2$ .

To explain (i) note that the change over the dimension of market power (while keeping information private)  $p_n - p_n^c$  inherits the order of the expected price difference  $E[p_n - p_n^c]$ , which is  $1/n$ . The change over the dimension of information (while fixing price-taking behavior)  $p_n^c - p_n^{fc}$  is of the order of  $\tilde{\theta}_n - \bar{\theta}$ , which is  $1/\sqrt{n}$ .<sup>7</sup> The interaction between both effects is easily seen to be of the same order as the market power term. This means that the difference  $p_n - p_n^{fc}$  will inherit the order of  $p_n^c - p_n^{fc}$  (information aggregation),  $1/\sqrt{n}$ , which is higher than the order of  $p_n - p_n^c$  (market power),  $1/n$ . A similar analysis establishes the same result for the alternative decomposition  $p_n - p_n^{fc} = p_n - p_n^f + p_n^f - p_n^{fc}$ . Now  $p_n - p_n^f$  is of the order  $1/\sqrt{n}$  (information aggregation) and  $p_n^f - p_n^{fc}$  is of the order  $1/n$  (market power).

The difference in expected prices between a strategic and a price-taking regime (be it with private information,  $p_n - p_n^c$ , or with full information,  $p_n^f - p_n^{fc}$ ) is of the order of  $1/n$ , which is the same as in markets with no uncertainty, and is explained by the rate at which market power vanishes. Indeed, in a full information Cournot equilibrium

$$\frac{p_n^f - \left(\sum_{i=1}^n MC(x_{in}^f, \theta_i)\right)/n}{p_n^f} = \frac{1/n}{\eta}$$

and the order of magnitude of the margin over the average marginal cost is  $1/n$  provided the elasticity of demand  $\eta$  is bounded away from zero and infinity.

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<sup>7</sup> We have that  $p_n - p_n^c = E[p_n - p_n^c] + (a_n - a_n^c)(\tilde{\theta}_n - \bar{\theta})$  and the second term is easily seen to be of lower order than  $1/n$ ;  $p_n^c - p_n^{fc} = (a_n^c - b_n^c)(\tilde{\theta}_n - \bar{\theta})$  and  $(a_n^c - b_n^c)$  is of the order of a constant.

The variance of the price difference keeping market power constant,  $p_n^c - p_n^{fc}$  or  $p_n - p_n^f$  is driven by information aggregation. That is, by the discrepancy between the sample mean and the population mean of the cost parameters,  $\tilde{\theta}_n - \bar{\theta}$ , which is of the order of  $1/\sqrt{n}$ . A firm in a private information regime has to estimate the market price (price-taking case) or residual demand (Cournot case), which depends on the average realization of the cost parameter, and his strategy will depend only on his cost realization (and the known population mean).

To explain result (ii) in the proposition consider the decomposition:

Deadweight loss = Information aggregation loss + Market power loss

$$(ETS_n^{fc} - ETS_n) / n = (ETS_n^{fc} - ETS_n^c) / n + (ETS_n^c - ETS_n) / n$$

or, alternatively,

$$(ETS_n^{fc} - ETS_n) / n = (ETS_n^f - ETS_n) / n + (ETS_n^{fc} - ETS_n^f) / n$$

which in terms of order of magnitude yields:

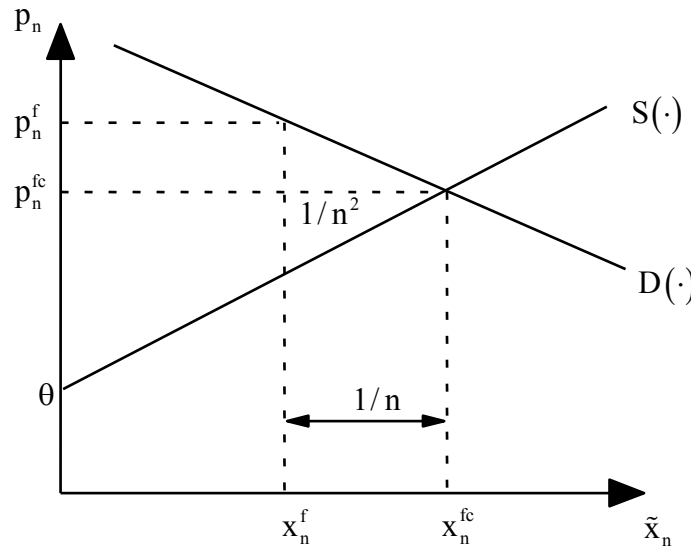
$$1/n \approx 1/n + 1/n^2$$

Market power dissipates according to the rate of the market without uncertainty. This yields an expected deadweight loss of the order of  $1/n^2$ , be it in the full information or in the private information cases. In the full information case (where we have  $(ETS_n^{fc} - ETS_n^f) / n$  this is a well-known result. The full information competitive equilibrium is determined by the intersection of aggregate (average) demand  $D(p) = \alpha - p$  with aggregate (average) supply  $S(p) = \lambda^{-1}(p - \tilde{\theta}_n)$ , yielding an average output of  $(\alpha - \tilde{\theta}_n) / (\lambda + 1)$ . Instead, the full information Cournot outcome yields an average output of  $(\alpha - \tilde{\theta}_n) / (\lambda + (1+n)/n)$ . The difference is of the order of  $1/n$  (see

e.g. Section 4.4 in Vives (1999)) and consequently the deadweight loss due to allocative inefficiency  $(1+\lambda)E\left[(x_n^f - x_n^{fc})^2\right]/2$  (the area of the “Harberger triangle”) is of the order of  $1/n^2$ . (See Figure 2.1.) To this we should add the productive inefficiency  $\lambda \text{var}\left[(x_{in}^{fc} - x_n^f) - (x_{in}^{fc} - x_n^{fc})\right]/2$  associated with not producing the Cournot output vector in a cost-minimizing way (inducing again a deadweight loss of the same order  $1/n^2$ ).<sup>8</sup>

Figure 2.1

Deadweight loss due to market power with full information



Similarly, in the private information case the order of both allocative and productive inefficiencies with respect to the Bayesian price-taking equilibrium (or decentralized team solution) is  $1/n^2$ . In both full and private information cases, market power affects the DWL not only by decreasing expected output (since  $b_n = b_n^f < b_n^c = b_n^{fc}$ ) but also by

<sup>8</sup> See the Appendix of the chapter for an explanation of the decomposition of the deadweight loss in allocative and productive inefficiency.

making firms less sensitive to their costs (respectively,  $a_n^f < a_n^{fc}$  and  $a_n < a_n^c$ ). Both allocative and productive inefficiency are impaired by the same order of magnitude.

When information aggregation is at stake, while keeping market power constant, the deadweight loss is driven by the variance terms because expected output does not change when switching from one regime to another. In a parallel fashion to Figure 2.1, we have, for example, that the discrepancy between  $x_n^c$  and  $x_n^{fc}$  is of the order of  $1/\sqrt{n}$  and, correspondingly, the allocative deadweight loss is of the order of  $1/n$ .<sup>9</sup>

Table 2.2 summarizes the order of magnitude of the welfare effects of price distortions.

		Deadweight loss due to:	
		Market power	Private information
Positive: Bayes-Cournot (ETS <sub>n</sub> )		$\Delta W_n^c = (ETS_n^c - ETS_n) / n$	$\Delta W_n^f = (ETS_n^f - ETS_n) / n$
Normative: Competitive (ETS <sub>n</sub> <sup>fc</sup> )		$\Delta \underline{W}_n^c = (ETS_n^{fc} - ETS_n^f) / n$	$\Delta \underline{W}_n^f = (ETS_n^{fc} - ETS_n^c) / n$
		order $1/n^2$	order $1/n$

**Table 2.2**  
Deadweight loss

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<sup>9</sup> We have that  $(ETS_n^{fc} - ETS_n^c) / n = ((\beta + \lambda) \text{var}[x_n^c - x_n^{fc}] + \lambda \text{var}[(x_{in}^c - x_n^c) - (x_{in}^{fc} - x_n^{fc})]) / 2$ . We know that  $x_n^c - x_n^{fc} = (b_n^c - a_n^c)(\tilde{\theta}_n - \bar{\theta})$ ,  $(b_n^c - a_n^c)$  is of the order of a constant and  $\text{var}[\tilde{\theta}_n - \bar{\theta}]$  is of the order  $1/n$ . Furthermore,  $(x_{in}^c - x_n^c) - (x_{in}^{fc} - x_n^{fc}) = a_n^{fc}(\theta_i - \tilde{\theta}_n) - a_n^c(\theta_i - \bar{\theta})$ , which again yields a productive efficiency loss of the order of  $1/n$  because of the leading term  $\tilde{\theta}_n$ .

### 2.4.3 How large is large: simulations

A natural question is how large a market should be so that the asymptotic (rate) results apply. Simulations with the model show that  $n$  need not be very large.<sup>10</sup> In the decomposition of  $DWL_n = ETS_n^{fc} - ETS_n$  in terms of  $\Delta W_n^f = (ETS_n^{fc} - ETS_n^c)$  and  $\Delta W_n^c = (ETS_n^c - ETS_n)$  there is a critical  $\bar{n}$  beyond which  $\Delta W_n^f > \Delta W_n^c$ . The critical  $\bar{n}$  decreases with  $\sigma_0^2$  and increases with  $\lambda$ . When uncertainty is significant and marginal costs are not very steep  $\bar{n}$  is below 10 firms. It may happen also that  $\bar{n}$  is below 2. The market power term  $\Delta W_n^c$  is typically always decreasing in  $n$ , while the information term  $\Delta W_n^f$  first increases and then decreases in  $n$ . This yields a total deadweight loss  $DWL_n$  which is, typically, always decreasing in  $n$ . Figure 2.2 displays the case with  $\alpha - \bar{\theta} = 6.5$ ,  $\lambda = 1/2$ , and  $\sigma_0^2 = 2$ , which yields  $\bar{n} = 4.4$ .<sup>11</sup>

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<sup>10</sup> Simulations have been performed in the following parameter range:  $\beta = 1$ ,  $\alpha - \bar{\theta}$  in  $\{2, 6.5\}$ , and  $\lambda$  and  $\sigma_0^2$  in  $\{1/10, 1/2, 1, 2, 10\}$ . Only  $\alpha - \bar{\theta}$  matters for the comparisons we perform. We get  $\alpha - \bar{\theta} = 2$  with, for example,  $\alpha = 3$ ,  $\bar{\theta} = 1$  and  $\alpha - \bar{\theta} = 6.5$  with, for example,  $\alpha = 10$ ,  $\bar{\theta} = 3.5$ . (Some of the parameter combinations in the simulations violate the sufficient conditions for positive outputs.)

<sup>11</sup> Similar results, with the possible exception of the comparative statics with respect to  $\lambda$  are obtained with the decomposition of  $DWL_n$  in terms of  $\Delta W_n^c = (ETS_n^{fc} - ETS_n^f)$  and  $\Delta W_n^f = (ETS_n^f - ETS_n)$ .

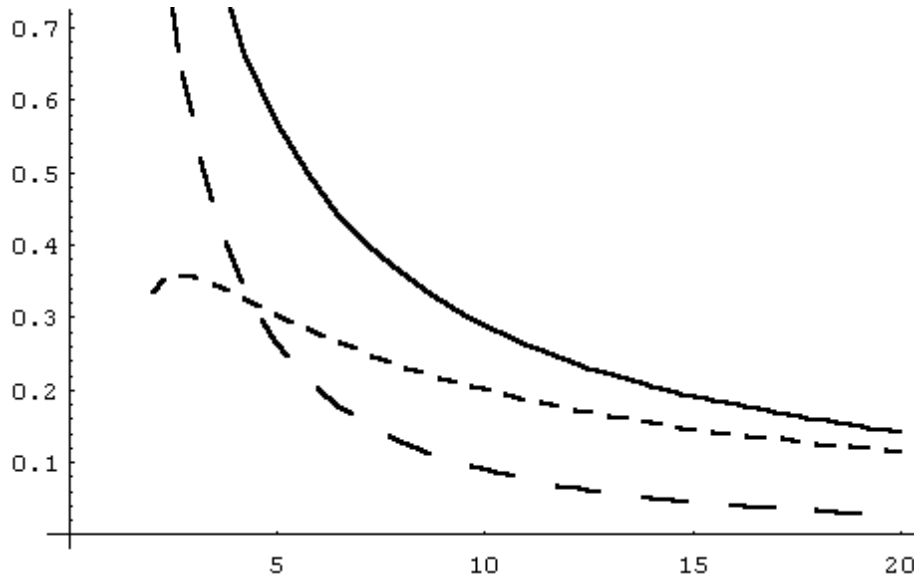


Figure 2.2

(Parameter values:  $\alpha - \bar{\theta} = 6.5, \lambda = 1/2, \sigma_0^2 = 2$ )

$$DWL_n = \Delta W_n^c + \Delta \underline{W}_n^f$$

( $DWL_n$  is the continuous line, the market power effect  $\Delta W_n^c$  is the dashed-discontinuous line, and the information effect  $\Delta \underline{W}_n^f$  is the discontinuous line)

#### 2.4.4 Summary

In an independent values Cournot market convergence to first best efficiency, as the market grows large, is attained at a slow rate because information aggregation is slow (at the rate  $1/\sqrt{n}$ ) and despite the fact that market power vanishes quickly (at the rate  $1/n$ ). Even for moderately sized markets we have that the lack of information aggregation looms larger than market power in the welfare loss with respect to the first best outcome.

#### 2.5 Convergence in auctions

We saw in Section 1.5 how auctions would aggregate information. In this section we will see that in auctions with independent valuations convergence to efficiency is faster than in the smooth Cournot market mechanism. That is, in an auction a smaller market is sufficient to get close to the full information outcome.

In Section 2.5.1 we survey results on the convergence to efficiency and price-taking behavior as the auction market grows large. Section 2.5.2 presents a procurement example where firms with random and independent costs bid for the right to face a downward sloping demand curve.

### 2.5.1 Convergence to efficiency and price-taking behavior

With independent valuations across bidders we may expect auction prices to converge to efficient prices as the number of bidders grows large. In particular, Swinkels (1999) shows that this is the case in a generalized first price auction. This is a discriminatory auction of several identical units of an object, where each bidder submits one or more bids, and each of the  $m$  highest bidders wins an object. The winners pay what they have bid. Asymptotic efficiency holds without any symmetry assumption among the bidders.

It would be interesting to know, however, the rate at which convergence to efficiency occurs. If convergence is fast then with a moderately sized market the standard (full information) competitive model provides a good approximation and the market outcome is approximately efficient. This is the case of the  $k$ -double auction with traders having independent valuations. At a  $k$ -double auction the bids of buyers and sellers are aggregated (respectively) into a demand and a supply schedule. Their intersection defines an interval of potential market clearing prices  $[\underline{p}, \bar{p}]$  from which  $(1-k)\underline{p} + k\bar{p}$  is selected with  $k \in [0,1]$ .

Rustichini, Satterthwaite, and Williams (1994) show that in this auction with  $n$  buyers and  $n$  sellers, each with unit demands and supplies, respectively, and independent reservation values, trader misrepresentation at symmetric, increasing, differentiable equilibria (and the discrepancy between the market price and the full information competitive price) is of the order of  $1/n$  and the corresponding inefficiency of the order of  $1/n^2$ . The authors do not establish the existence of equilibrium in the double auction. This is done by Jackson and Swinkels (2005) in a private value environment. Satterthwaite and Williams (2002) establish that this rate of convergence is the fastest

among all mechanisms in the i.i.d. environment.<sup>12</sup> Cripps and Swinkels (2006) extend the results to a generalized private value setting where bidders can be asymmetric and can demand or supply multiple units. Under some regularity conditions and a weak requirement of “a little independence” (amounting for each player to have a small idiosyncratic component in his valuation), the authors find that as the number of players  $n$  grows all nontrivial equilibria of the double auction converge to the competitive outcome and inefficiency vanishes at the rate of  $1/n^{2-\alpha}$  for any  $\alpha > 0$ . In short, with few traders the outcome of the  $k$ -double auction is close to the competitive one. This contrasts with the slower rate of convergence of the Cournot market mechanism with independent costs.

Gong and McAfee (1996) extend the work on double auctions by Rustichini et al. (1994) to general information structures. Gong and McAfee show that either with private values (where agents receive perfect signals about their types and the types are allowed to be correlated) or when buyers and sellers are symmetric (in terms of utility functions and distributions) the price-taking equilibrium maximizes expected total surplus under the constraint of using decentralized strategies. Then the discrepancy of the strategic equilibrium (Bayesian bidding equilibrium at the double auction) and the non-strategic (price-taking) equilibrium bid is of the order of  $1/n$  and the associated expected inefficiency of the order of  $1/n^2$ .<sup>13</sup> These results are consistent with those obtained in the Cournot setting with a general information structure (see Proposition 2.2).

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<sup>12</sup> In the symmetric  $k$ -double auction the efficiency issue is whether the number of objects being sold is the right one, while in the asymmetric auctions considered by Swinkels (1999) the efficiency question is whether the objects go to the bidders with the highest valuations. See also Satterthwaite and Williams (1989) and Williams (1991).

<sup>13</sup> The result may not hold if buyers and sellers are asymmetric. Then the social planner takes into account externalities between buyers and sellers which individual non-strategic agents do not care about. The situation is akin to a noisy rational expectations model, in which the trade of an agent creates an information externality for others (see Chapter 3).

An interesting application to a Internet auction environment similar to e Bay is presented by Peters and Severinov (2006). The authors consider a decentralized trading mechanism where sellers with a unit of a homogeneous good who set reserve prices for their independently run second-price auctions. Each buyer has a unit demand, has a private value for the good and can bid and move between auctions. A perfect Bayesian equilibrium, with a simple strategy, is characterized. It is found that in a large finite market, and under the assumption that the valuation of a trader takes each positive value with positive probability, sellers post reserve prices equal to their costs.

### 2.5.2 Price competition with independent constant marginal costs

Consider now price competition in a homogenous product market with constant marginal costs, which are identically and independently distributed on a compact interval with positive continuous density. In this case sellers bid for the right to supply the market, that is, to face the downward sloping demand curve.<sup>14</sup> (With constant marginal costs the replication of demand only introduces a scale factor which we will ignore here.) The Bayesian Bertrand equilibrium of this price competition model has been considered by Hansen (1988) and Spulber (1995). The firm that quotes the lowest price gets all the market (if more than one firm quotes the lowest price they share the market). It is well-known that with complete information that the equilibrium price equals the cost of the second more efficient firm and there is no productive inefficiency because the lowest cost firm supplies the market.

With incomplete information it can be shown (see Spulber (1995)) that there is a unique and symmetric Bayesian Bertrand equilibrium  $p_n(\theta)$ , which is increasing and differentiable. The equilibrium strategy is given by the solution to the following differential equation, with an appropriate boundary condition,

$$p_n(\theta) - \theta = p'_n(\theta) D(p_n(\theta)) / \left( (n-1) h(\theta) D(p_n(\theta)) - p'_n(\theta) D'(p_n(\theta)) \right),$$

---

<sup>14</sup> With increasing marginal costs we should worry about rationing schemes for unsatisfied demand where firms refuse to serve customers at the quoted price and things get complicated.

where  $h(\theta)$  is the hazard rate of the distribution (the probability that the cost of a rival equals  $\theta$  given that it is not less than  $\theta$ ). Under standard boundedness conditions  $p_n(\theta) - \theta$  will be of the order of  $1/n$  and, therefore, so will be the margin of the winning firm  $p_n(\theta_{\min}) - \theta_{\min}$ .

Given that  $p_n(\theta)$  is increasing there is no productive inefficiency, and the lowest cost firm supplies the market. Furthermore, the deadweight loss due to allocative inefficiency is of the order of  $1/n^2$ . The winner-takes-all nature of the competition implies that the equilibrium strategy depends on  $(n-1)h(\theta)$ , that is, on the probability that, conditional on having the lowest cost, a rival has also the lowest cost. The higher is the probability the lower is the margin, ceteris paribus. The firm conditions on its cost realization and on the event of winning the contest. The consequence is that in the Bertrand game a firm is effectively conditioning on more information than in the Cournot game. In the latter case a firm has to estimate the average cost of (active) rivals and relies only on its cost realization. The auction/Bertrand mechanism aggregates information better than the Cournot mechanism and this explains the different rates at which prices converge to marginal cost in the presence of incomplete information. In Chapter 3 we will study the information aggregation properties of supply function competition where firms can condition on the market price.

### 2.5.3 Summary

Equilibrium prices in auctions with independent values converge fast to the full information competitive price (at the rate of  $1/n$ , where  $n$  is the size of the market, be it the common number of buyers and sellers in a  $k$ -double auction or the number of bidders in a first price auction). Correspondingly, the deadweight loss is of the order of  $1/n^2$ . This contrast with Cournot markets, where the rates are, respectively,  $1/\sqrt{n}$  and  $1/n$ .

## **Summary**

In this chapter we have introduced strategic behavior and have looked at large Cournot and auction markets, which induce behavior close to competitive. We have checked whether the continuum model of Chapter 1 is a good approximation in a large market and how large a market should be for the approximation to be good. In other words, we have looked at the convergence properties as markets grow large and potentially price-taking behavior and information aggregation are obtained. The results are as follows.

In a general Cournot market:

- Equilibria of the continuum economy (derived in Chapter 1) are found to be the limits of equilibria of finite economies and the convergence rate is characterized. Moreover, convergence is faster with common values.
- Convergence to price taking behavior is fast (at the rate of  $1/n$  where  $n$  is the “size” of the market) and the associated deadweight loss vanishes quickly (at the rate of  $1/n^2$ ).

With independent values:

- Both Cournot and auction markets aggregate information, and as the market grows large an efficient outcome is obtained.
- However, convergence is faster in the auction market because a Cournot market aggregates information slowly (at the rate of  $1/\sqrt{n}$  and this slow convergence dominates, even in moderately sized markets, the faster dissipation of market power).
- The deadweight loss in the Cournot market is of the order of  $1/n$  while in an auction is of the order of  $1/n^2$ .

## **Appendix**

Proposition 2.1. In the linear-normal model there is a unique Bayesian Cournot equilibrium and a unique price-taking Bayesian Cournot equilibrium. They are symmetric, and affine in the signals. The strategies of the firms are given (respectively) by:

$$X_n(s_i) = b_n(\alpha - \bar{\theta}) - a_n(s_i - \bar{\theta}), \quad a_n = \frac{\xi}{(2\beta/n) + \lambda + \beta\zeta\xi((n-1)/n)},$$

$$b_n = \frac{1}{\lambda + \beta((1+n)/n)}$$

$$X_n^c(s_i) = b_n^c(\alpha - \bar{\theta}) - a_n^c(s_i - \bar{\theta}), \quad a_n^c = \frac{\xi}{(\beta/n) + \lambda + \beta\zeta\xi((n-1)/n)}, \quad b_n^c = \frac{1}{\lambda + \beta}.$$

Proof: Drop the subscript  $n$  labeling the replica market and let  $\beta = 1$  to ease notation. We consider first the Bayesian Cournot equilibrium. We check that the candidate strategies form an equilibrium. The expected profits of firm  $i$  conditional on receiving signal  $s_i$  and assuming firm  $j, j \neq i$ , uses strategy  $X_j(\cdot)$  are

$$E[\pi_i | s_i] = x_i \left( \alpha - E[\theta_i | s_i] - \frac{1}{n} \sum_{j \neq i} E[X_j(s_j) | s_i] - \left( \frac{1}{n} + \frac{\lambda}{2} \right) x_i \right).$$

Then first order conditions (F.O.C.) yield

$$2 \left( \frac{1}{n} + \frac{\lambda}{2} \right) x_i(s_i) = \alpha - E[\theta_i | s_i] - \frac{1}{n} \sum_{j \neq i} E[X_j(s_j) | s_i], \text{ for } i = 1, \dots, n.$$

Plugging in the candidate equilibrium strategy and using the formulae for the conditional expectations for  $E[\theta_i | s_i]$  and  $E[s_j | s_i]$ , (see Section 2 in the Technical Appendix)

$$E[\theta_i | s_i] = \xi s_i + (1 - \xi)\bar{\theta} \quad \text{and} \quad E[s_j | s_i] = E[\theta_j | s_i] = \xi\zeta s_i + (1 - \xi\zeta)\bar{\theta},$$

it is easily checked that they satisfy the FOC (which are also sufficient in our model). To prove uniqueness (1) we show that the Bayesian Cournot equilibria of our game are in one-to-one correspondence with person-by-person optimization of an appropriately defined concave quadratic team function; (2) we note that person-by-person optimization is equivalent in our context to the global optimization of the team function (since the random term does not affect the coefficients of the quadratic terms and the team function is concave in actions, Radner (1962, Theorem 4)); and (3) we invoke the result by Radner,

which implies that in our linear-quadratic model with the type of uncertainty considered and jointly normal random variables, the components of the unique Bayesian team decision function of the equivalent team problem are affine (Radner (1962, Theorem 5)). Based on the above three observations we conclude that the affine Bayesian Cournot equilibrium is the unique equilibrium.<sup>15</sup>

Let us show (1) by displaying an appropriate team function  $G$ . A team decision rule  $(X_1(s_1), \dots, X_n(s_n))$  is person-by-person optimal if it can not be improved by changing only one component  $X_i(\cdot)$ . (This just means that each agent maximizes the team objective conditional on his information and taking as given the strategies of the other agents.) Let  $G(x) = \pi_i(x) + f_i(x_{-i})$  where

$$f_i(x_{-i}) = \sum_{j \neq i} (\alpha - \theta_j) x_j - \left( \frac{1}{n} + \frac{\lambda}{2} \right) \sum_{j \neq i} x_j^2 - \frac{1}{2n} \sum_{\substack{k \neq j \\ k, j \neq i}} x_k x_j.$$

This yields

$$G(x) = \sum_j (\alpha - \theta_j) x_j - \left( \frac{1}{n} + \frac{\lambda}{2} \right) \sum_j x_j^2 - \frac{1}{2n} \sum_{i \neq j} x_i x_j.$$

We obtain the same outcome by solving either  $\max_{x_i} E[\pi_i | s_i]$  or  $\max_{x_i} E[G | s_i]$  since  $f_i(x_{-i})$  does not involve  $x_i$ .

A similar argument establishes the result for the price-taking equilibrium. Then the F.O.C. for firm  $i$  is given by

$$\lambda x_i(s_i) = \alpha - E[\theta_i | s_i] - \frac{1}{n} \sum_j E[X_j(s_j) | s_i],$$

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<sup>15</sup> This method of showing uniqueness of Bayesian Cournot equilibrium in linear-quadratic models with normal distributions (or more in general, with affine information structures) has been used by Basar and Ho (1974) and Vives (1988).

and the solution is a (person-by-person) maximum of a team problem with an objective function which is precisely the ETS as shown in Proposition 1.2. ♦

In order to perform welfare comparisons and prove propositions 2.2 and 2.4 we will need the following Lemma.

Lemma. The difference in (per capita) ETS between a price-taking regime R and another regime with strategies based on less information (that is, on a weakly coarser information partition) is given by

$$(ETS^R - ETS)/n = \left( \beta E \left[ (x_n - x_n^R)^2 \right] + \lambda \left( \sum_{i=1}^n E \left[ (x_{in} - x_{in}^R)^2 \right] / n \right) \right) / 2.$$

The result follows considering a Taylor series expansion of TS (stopping at the second term due to the quadratic nature of the payoff) around price-taking equilibria. The key to simplify the computations is to notice that at price-taking equilibria total surplus is maximized. Note that if the strategies and the information structure are symmetric then

$$E \left[ (x_{in} - x_{in}^R)^2 \right] \text{ is independent of } i \text{ and therefore } \sum_{i=1}^n E \left[ (x_{in} - x_{in}^R)^2 \right] / n = E \left[ (x_{in} - x_{in}^R)^2 \right].$$

Allocative and productive inefficiency. We can decompose the total inefficiency with respect to price-taking regime R in allocative and productive inefficiency. The latter is associated to the production of an average output in a non cost-minimizing way. The former to the loss in surplus when producing, in a cost-minimizing way, an average output different from the benchmark. Consider a symmetric information structure and strategies. When average outputs  $x_n$  and  $x_n^R$  are produced in a cost minimizing way then for all  $i$ ,  $x_{in} - x_{in}^R = x_n - x_n^R$ .<sup>16</sup> This implies that pure allocative inefficiency is given by

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<sup>16</sup> An immediate implication of cost minimization when producing average output  $x$  is that  $x_i = x + (\tilde{\theta}_n - \theta_i) / \lambda$ .

$(\beta + \lambda)E\left[(x_n - x_n^R)^2\right]/2$ . The residual is due to productive inefficiency and can be expressed as  $\lambda E\left[(u_i - v_i)^2\right]/2$  where  $u_i = x_{in} - x_n$  and  $v_i = x_{in}^R - x_n^R$ . We can write thus  $(ETS^R - ETS)/n = \left((\beta + \lambda)E\left[(x_n - x_n^R)^2\right] + \lambda E\left[(u_i - v_i)^2\right]\right)/2$ . Note that  $E\left[(u_i - v_i)^2\right] = \text{var}[u_i - v_i]$  because  $E[u_i] = E[v_i] = 0$ .

Proposition 2.2. As the market grows large the market price (at the Bayesian Cournot equilibrium)  $p_n$  converges in mean square to the price-taking Bayesian price  $p_n^c$  at the rate of  $1/n$ . (That is,  $E\left[(p_n - p_n^c)^2\right]$  tends to 0 at the rate of  $1/n^2$ .) The (per capita) expected deadweight loss at the market outcome with respect to Bayesian price-taking equilibria  $(ETS_n^c - ETS_n)/n$  is of the order of  $1/n^2$ .

Proof: Consider wlog the case  $\beta = 1$ . Let  $y_n = p_n - p_n^c = x_n^c - x_n = (b_n^c - b_n)(\alpha - \bar{\theta}) + (a_n - a_n^c)(\tilde{s}_n - \bar{\theta})$ . Recall that  $E\left[(y_n)^2\right] = (E[y_n])^2 + \text{var}[y_n]$ . We have that  $E[y_n] = (b_n^c - b_n)(\alpha - \bar{\theta})$  because  $E[\tilde{s}_n] = \bar{\theta}$ . It is easily seen that  $(b_n^c - b_n)$  is of order  $1/n$  (indeed,  $n(b_n^c - b_n)$  tends to  $1/(1 + \lambda)^2$  as  $n$  tends to infinity). Therefore  $(E[y_n])^2$  is of order  $1/n^2$ . Furthermore,  $\text{var}[y_n] = (a_n - a_n^c)^2 \text{var}[\tilde{s}_n]$ . We have that  $\text{var}[\tilde{s}_n] = \left((1 + (n-1)\zeta)\sigma_\theta^2 + \sigma_\varepsilon^2\right)/n$ , which is of the order of a constant for  $\zeta > 0$  (or  $1/n$  for  $\zeta = 0$ ), and that  $(a_n - a_n^c)$  is of order  $1/n$  (because  $n(a_n - a_n^c)$  tends to  $-\xi(\zeta\xi + \lambda)^{-2}$  as  $n$  tends to infinity). Therefore the order of  $\text{var}[y_n]$  is  $1/n^2$  for  $\zeta > 0$  (or  $1/n^3$  for  $\zeta = 0$ ). We conclude that in any case the order of  $y_n = p_n - p_n^c$  is  $1/n$ . Consider  $(ETS_n^c - ETS_n)/n$  now. According to the Lemma above and given that equilibria are symmetric we have that

$(ETS_n^c - ETS_n)/n = (\beta E[(x_n - x_n^c)^2] + \lambda E[(x_{in} - x_{in}^c)^2])/2$  . We have just shown  $E[(x_n - x_n^c)^2]$  to be of order  $1/n^2$ . We have that  $E[(x_{in} - x_{in}^c)^2] = (E[x_{in} - x_{in}^c])^2 + \text{var}[x_{in} - x_{in}^c]$  . Now,  $E[x_{in} - x_{in}^c]$  is of the same order as  $E[x_n - x_n^c]$ ,  $1/n$ , and  $\text{var}[x_{in} - x_{in}^c] = (a_n - a_n^c)^2 (\sigma_\theta^2 + \sigma_\varepsilon^2)$ , is of order  $1/n^2$  because  $(a_n - a_n^c)$  is of order  $1/n$ . Therefore,  $E[(x_{in} - x_{in}^c)^2]$  is of order  $1/n^2$ . We conclude that  $(ETS_n^c - ETS_n)/n$  is of the order of  $1/n^2$ . ♦

Proposition 2.3. As the economy is replicated the Bayesian Cournot equilibrium converges to the equilibrium in the continuum limit economy:  $X(s_i) = b(\alpha - \bar{\theta}) - a(s_i - \bar{\theta})$ , where  $a = \xi / (\lambda + \beta \zeta \xi)$  and  $b = 1 / (\lambda + \beta)$ . The Bayesian Cournot price  $p_n$  converges (in mean square) to  $p = \alpha - \beta(b(\alpha - \bar{\theta}) - a(\tilde{\theta} - \bar{\theta}))$  at the rate of  $1/\sqrt{n}$  and  $\sqrt{n}(p_n - p)$  converges in distribution to  $N(0, \beta^2 a^2 ((1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2))$ .

Proof: For the first part, from Proposition 2.1 we have that

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{\xi}{(2\beta/n) + \lambda + \beta \zeta \xi ((n-1)/n)} = \frac{\xi}{\lambda + \beta \zeta \xi} = a$$

and

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{\lambda + \beta((1+n)/n)} = \frac{1}{\lambda + \beta} = b.$$

Hence,

$$\lim_{n \rightarrow \infty} X_n(s_i) = \lim_{n \rightarrow \infty} [b_n(\alpha - \bar{\theta}) - a_n(s_i - \bar{\theta})] = b(\alpha - \bar{\theta}) - a(s_i - \bar{\theta}) = X(s_i).$$

For the second part of the proposition note that  $x_n = b_n(\alpha - \bar{\theta}) - a_n(\tilde{s}_n - \bar{\theta})$ . We have that

$$\begin{aligned} p_n &= \alpha - \beta [b_n(\alpha - \bar{\theta}) - a_n(\tilde{s}_n - \bar{\theta})] \\ p &= \alpha - \beta [b(\alpha - \bar{\theta}) - a(\tilde{\theta} - \bar{\theta})] \end{aligned}$$

and  $E[p_n - p] = \beta(b - b_n)(\alpha - \bar{\theta})$  since  $E[\tilde{s}_n] = E[\tilde{\theta}] = \bar{\theta}$ . Note that  $(b - b_n)$ , and therefore  $E[p_n - p]$ , tends to 0 as  $n$  tends to infinity. Since  $\text{var}[p_n - p] = E[(p_n - p)^2] - (E[p_n - p])^2$  to conclude that  $\lim_{n \rightarrow \infty} E[(p_n - p)^2] = 0$  it is sufficient to show that  $\lim_{n \rightarrow \infty} \text{var}[\sqrt{n}(p_n - p)] = \beta^2 a^2 ((1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2)$ . From which it follows that  $\sqrt{n}(p_n - p)$  converges in distribution to  $N(0, \beta^2 a^2 ((1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2))$  because  $p_n - p$  is normally distributed. We have that

$$\text{var}[\sqrt{n}(p_n - p)] = n\beta^2 \text{var}[a_n \tilde{s}_n - a\bar{\theta}] = \beta^2 \frac{\{a_n^2 n^{-1} [\sigma_\theta^2 (1 + \zeta(n-1)) + \sigma_\varepsilon^2] + a\zeta\sigma_\theta^2 (a - 2a_n)\}}{n^{-1}}$$

Using L'Hôpital's rule and the fact that  $\partial a_n / \partial n$  is of the order of  $1/n^2$  we obtain

$$\begin{aligned} \lim_{n \rightarrow \infty} \text{var}[\sqrt{n}(p_n - p)] &= \\ \beta^2 \lim_{n \rightarrow \infty} &\frac{\left\{ 2a_n \frac{\partial a_n}{\partial n} \left[ \frac{(1 + \zeta(n-1))\sigma_\theta^2}{n} + \frac{\sigma_\varepsilon^2}{n} \right] + a_n^2 \left[ -\frac{(1 - \zeta)\sigma_\theta^2}{n^2} - \frac{\sigma_\varepsilon^2}{n^2} \right] - 2 \frac{\partial a_n}{\partial n} a\zeta\sigma_\theta^2 \right\}}{-\frac{1}{n^2}} \\ &= \beta^2 a^2 [(1 - \zeta)\sigma_\theta^2 + \sigma_\varepsilon^2]. \blacklozenge \end{aligned}$$

## Exercises

**2.1 Optimality of information acquisition under constant returns.** Show in the endogenous information acquisition model of Section 2.3 that if returns are constant then as  $n$  tends to infinity the subgame-perfect equilibrium outcome of the two-stage game attains the first best.

*Solution:* With constant returns to scale the equilibrium output strategies given a common precision  $\xi$  are given by  $X_n(s_i) = a_n(s_i - \bar{\theta}) + b\bar{\theta}$  with  $a_n = \frac{n\xi}{\beta(2 + (n-1)\xi)}$

and  $b = \frac{1}{2\beta}$ . Substituting in the expression for per capita total gross surplus and taking

expectations, we obtain:  $EGS_n = \frac{n(2+n)}{2(1+n)^2} \bar{\theta}^2 + \frac{n\xi((n-1)\xi+3)}{((n-1)\xi+2)^2} \frac{\sigma_\theta^2}{2\beta}$ . Therefore, per

capita expected total surplus at the market equilibrium  $\xi_n^*$  is  $ETS_n = EGS_n - \frac{c}{\sigma_\theta^2} \frac{\xi_n^*}{1-\xi_n^*}$ .

As  $n$  tends to infinity  $\xi_n^*$  converges to zero and  $n\xi_n^*$  to infinity. This implies that  $ETS_n \xrightarrow{n} (\bar{\theta}^2 + \sigma_\theta^2) / 2\beta$ , which is the first best level.

**2.2 Measuring the Harberger triangle.** A typical empirical assessment of the deadweight loss due to market power uses an approximation at the industry level with data on profit returns and sales positing a certain value for the elasticity of demand (under constant returns the deadweight loss is approximated by  $R\eta d^2 / 2$ , where  $R$  is revenue,  $\eta$  the demand elasticity and  $d$  the relative price distortion from the competitive level). Harberger and others have obtained low estimates for the welfare loss (Scherer and Ross (Ch. 18, 1990)). Consider a Cournot market with uncertain costs which are private information to the firms as in Section 2.4. What are the implications of the analysis in the section for the estimation of the Harberger triangle?

*Answer:* Harberger's small estimates should not be surprising from the perspective of the results in Section 2.4 because in moderately sized markets the effect of market power may be relatively small. However, accounting for incomplete information would increase the deadweight loss estimate. According to our model the true deadweight loss due to allocative inefficiency is given by  $(\beta + \lambda) \left[ \left( E[x_n - x_n^{fc}] \right)^2 + \text{var}[x_n - x_n^{fc}] \right] / 2$ . The Harberger approach would approximate it by the first term in the sum, which is of order  $1/n^2$  while the second is of order  $1/n$ . Under full information the approximation is fine (because then  $\text{var}[x_n^f - x_n^{fc}]$  is of order  $1/n^3$ ) but not so with private information. Furthermore, to the allocative inefficiency measure the loss due to productive inefficiency  $(\lambda \text{var}[(x_{in} - x_n) - (x_{in}^{fc} - x_n^{fc})]) / 2$  should be added. The deadweight loss would be higher by an order of magnitude. For example, with a market concentration

equivalent of ten firms ( $n = 10$ ), while the true relative deadweight loss is of the order of 10% the estimated one ignoring private information would be of the order of only 1%.

2.3 Welfare with price competition and private information. Examine the welfare properties of the procurement auction model of Section 2.5.2. Compare in particular expected total surplus with full information and with private information.

*Answer:* The deadweight loss is larger under full than under incomplete information. Contrary to the Cournot case, we have that  $ETS_n^f < ETS_n$ . The reason is that with incomplete information a firm tends to be more aggressive and sets a price below the expected price with complete information. Denote by  $\theta_{next}$  the cost of the second most efficient firm. The result is that  $p(\theta_{min}) < E[\theta_{next} | \theta_{min}]$ . Indeed, with inelastic demand the revenue equivalence theorem would apply (think of Bertrand competition with complete information as an open descending auction) and we would have that  $p(\theta_{min}) = E[\theta_{next} | \theta_{min}]$ . This balances exactly the fact that in the first price auction raising the bid a little bit increases profit when winning but reduces the probability of winning. If demand is elastic then the profit increase when winning is strictly smaller and therefore the optimum bid must be smaller than before. The fact that incomplete information may improve welfare should not be surprising, because we are in a second best world there is no guarantee that welfare increases by moving towards full information while maintaining market power. The Cournot model is more “regular” and lifting one distortion at a time improves welfare (see Hansen (1988) and Vives (2002)).

2.4 Competition policy implications. What policy implications can you draw from the relative weights of market power and private information as sources of welfare loss? Does your answer depend on the type of competition in the market?

*Hint:* Speculate on the basis of the results in the chapter.

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