Private Label Introduction: Does it Benefit the Supply Chain?  

Marc Sachon • Victor Martínez-de-Albéniz

IESE Business School, University of Navarra, Av. Pearson 21, 08034 Barcelona, Spain

msachon@iese.edu • valbeniz@iese.edu


Abstract

Private labels have changed the retail industry in recent years. They are used by retailers to put pressure on manufacturer pricing. In this paper, we investigate the impact of private label introduction on supply chain profits, i.e., the profits achieved by the entire channel composed of one retailer and several manufacturers. We propose three supply chain games with linear demand to compare the situation without private label to one where a private label replaces an brand or is added to the assortment. We find that system efficiency may be reduced when product substitutability is high in the replacement case, and when it is not too low nor too high in the addition case, because the retailer drives too much demand to the private label. Our results extend to general demands under certain regularity conditions. Our findings suggest that when a category is sufficiently competitive, private label introduction may destroy value and retailers and manufacturers can find better ways to share profits.

Keywords: Private label, non-cooperative game theory, supply chain efficiency.

1 Introduction

Private labels, also known as store brands, account for at least 40% of all products sold in Germany, the UK and Spain (PLMA 2011), and 24% in the United States (PLMA 2010). These are products that are specific to a retail chain and cannot be bought at competing retailers. They are controlled by the retailer who has exclusive rights on them and may be manufactured directly by the retail chain (e.g., coffee at Aldi, the German hard discounter) or a third-party manufacturer, which may only produce private labels, e.g., Cott of Canada in soft drinks, or also produce a brand with its own name, e.g., Friesland-Campina of the Netherlands in dairy products.

Private labels in groceries have been around for more than half a century but only recently have we seen a significant growth of their market share: total sales of private label grocery products in Europe reached 100 billion euro sales in 2005 for the first time, and have outpaced manufacturer brands in more than half of the markets measured (ACNielsen 2005). Even global brand leaders such as Kellogg and Unilever are affected by this development: “Retailers are

1Research supported in part by the international logistics research center (CIIL) and the sector público - sector privado research center (SPSP) through contract ECO 2008 05155, ECON, both of IESE Business School, University of Navarra.
increasingly offering private label products that compete with our products” (Kellogg 2009);
“In France, Spain and Germany markets were difficult, with branded products losing ground to private label.” (Unilever 2009). Industry experts such as PlanetRetail even go further: a recent report is entitled “Private Label: The Brands of the Future” (PlanetRetail 2010).

Private labels allow retailers to increase sales of a category, because they are priced competitively and increase the variety offered to the consumer. For example, Sainsbury and Waitrose in the UK put emphasis on developing their own private labels and increased store sales from 2008 to 2010 in a contracting market (Coase 2009). They are also a tool to generate shopper loyalty (Thomassen et al. 2006), and have the additional benefit of very low marketing and sales costs (e.g., no costly advertising campaigns or end-of-aisle displays), resulting in a lower cost structure. Last but not least, retailers use private labels to gain leverage in their relationships with leading brand manufacturers. As stated by GMA et al. (2007), “retailer strategies and capabilities are driving the evolution of private labels”.

The academic literature has shown that private labels are in fact an instrument to put pressure on brand manufacturers, which react by reducing wholesale prices, see Raju et al. (1995). It is hence not surprising that brand manufacturers suffer from these introductions. However, some of the manufacturers put forward a more serious argument: they complain that private labels are not only changing the balance of power in the channel, but also destroying value for the entire supply chain, composed of retailer and all manufacturers. Specifically, they claim that consumers that would be willing to pay more for a product are switching to the private label, and thereby overall profitability is reduced. If true, this concern would imply that there may be Pareto-improving alternatives to the use of private label, which are especially relevant in the retail industry, where margins are razor-thin. In other words, private label is used to transfer rents from manufacturers to retailers, with a net loss for the retail chain: one could think of better ways to achieve the same rent transfer without diminishing the total supply chain profit. To investigate whether such concerns are well founded is precisely the objective of this paper.

For this purpose, we study three supply chain games with linear demand. We compare a benchmark scenario where two manufacturers compete, to (i) a scenario where a private label replaces one of the manufacturers, and to (ii) a scenario where a private label is added to the existing assortment. We confirm that private is indeed always beneficial for the retailer and detrimental to the manufacturers that need to compete with the private label. More importantly, we find that total supply chain profit may increase or decrease. On the one hand, the private label reduces double marginalization in the prices of the branded products. Double marginalization is a prevalent phenomenon in supply chain management, studied since Spengler
(1950), that results in higher prices than what would be optimal for the entire supply chain due to the lack of pricing coordination between manufacturer and retailer. When introducing a private label, the retailer de facto reduces the retail price of one of the products in the category, which forces the manufacturer to respond by reducing the wholesale price of its competing product, which reduces the double marginalization effect. On the other hand, there may be too many customers switching from manufacturer brand to the private label, which may generate higher margin for the retailer, but lower margin for the supply chain. This reduces system profit. The balance of the two effects determines the effect on supply chain efficiency.

We find that the key driver of this balance is product substitutability, i.e., the sensitivity of one’s price to a competitor’s sales quantity compared to the sensitivity to one’s own sales quantity. We show that higher substitutability is associated with worse supply chain performance when the private label replaces a brand. In contrast, when the private label is added, due to the market expansion associated with the larger assortment, supply chain performance deteriorates when product substitutability is neither too low nor too high.

We then extend the model to a more general situation with nonlinear demand and a general number of players. We provide sufficient conditions under which the three scenarios can be analyzed, and discuss the robustness of our insights. In particular, we show that when the demand follows the multinomial logit (MNL) choice model, then a unique equilibrium always exists, in which private label introduction always benefits retailer and hurts manufacturers. We observe that supply chain profit may decrease when the scenario without private label is sufficiently competitive, e.g., when there are many manufacturers. Our findings are hence robust. We can thus use the insights of our model to build recommendations on the use of private labels in retail chains: when a category already has strong competition, even though retailer still benefits from a private label introduction, manufacturers and retailers should consider alternatives that preserve supply chain efficiency. For instance, manufacturers should prevent private label introductions by lowering wholesale prices before it is too late.

In terms of contributions, our paper is one of the first to analyze the impact of private labels on system efficiency, and characterize product substitutability as the key determinant to the variation of supply chain profits. In addition, from a methodological standpoint, we provide conditions for the existence and uniqueness of supply chain pricing equilibrium even when demand is nonlinear, and we show that they are satisfied for MNL demand.

The rest of the paper is organized as follows. We review the related literature in §2. We then present our basic game-theoretic model with two products and linear demand in §3, where we discuss the impact of a private label which replaces an existing brand or is added to the assortment. §4 shows how to apply our model to more general assumptions on customer be-
behavior (nonlinear demand) and the competitive setting (any number of manufacturers in the category). We conclude the paper with recommendations in §5.

2 Literature Review

Our work is related to the literature on supply chain games and in particular on private labels, mostly in marketing and operations management.

First, there is a broad stream of research that studies the impact of channel structure on prices and market shares. Jeuland and Shugan (1983) discuss the coordination problem in a decentralized channel, and in particular provide an analysis where there may be competitive reactions from other retailers. In a two-retailers two-manufacturers chain, McGuire and Staelin (1983) analyze when a certain distribution structure (franchises vs. manufacturer-owned stores) is preferred over others by manufacturers. Specifically, they show that for high product substitutability, decentralized distribution yields higher profits. We make a similar finding with one single retailer, but the driver in our case is that private label may introduce asymmetry across products thereby reducing supply chain profits. Following McGuire and Staelin (1983), Shugan and Jeuland (1988) discuss the vertical (within the channel) and horizontal (between channels) dimensions of competition. Choi (1991) studies price dynamics of two competing national brand manufacturers that sell through a common retailer, as we do. He shows that the sequence of decision-making matters, through the analysis of three different non-cooperative games (retailer Stackelberg, manufacturer Stackelberg and vertical Nash), while we focus on showing how a private label impacts the equilibrium profits. Also, Lee and Staelin (1997) analyze vertical strategic interactions between channel players and its effect on channel pricing strategies. They find situations in which a channel member can be better off by not using foresight of the other channel member’s reaction if the latter also has uses foresight in making the pricing decision. Trivedi (1998) analyzes the effect of different channel structures (integrated, decentralized and full distribution channel) on profits and price, under a symmetric channel structure.

There is also much work on private labels. Kumar and Steenkamp (2007) provide a broad overview of the field. Most of the papers focus on when and how to introduce a store brand. In this line of research, Raju et al. (1995) provide an exhaustive analysis of what makes a product category more conducive for private label introduction. They highlight that categories with initially low competition between national brands are the best suited for store brand introduction, and identify cross-price sensitivity between national and store brand as a key driver of the retailer’s profit increase. Sethuraman et al. (1999) confirm empirically that brands that are priced at a similar level tend to have higher cross-price sensitivities. They provide an
extensive empirical investigation on cross-price elasticities between brands and private labels. Cotterill et al. (2000) also study the interaction between national brands and private labels and focus on considering competitor price responses in their empirical analysis. Sayman et al. (2002) analyze retailer decisions on the positioning of a store brand vs. two national brands. They support their findings with an empirical study and show that using a store brand to target the leading national brand increases the retailer’s profits at the cost of the targeted manufacturer. They suggest that a store brand can alleviate the double marginalization problem, but we show that this is not always true. The impact of quality differences between products has also been considered, through Hotelling models. Narasimhan and Wilcox (1998) analyze a retailer’s strategic use of private label as a means of obtaining better terms of trade from national brand manufacturers, and empirically validate their results. Chen et al. (2008) study the effect that developments costs and differentiated marginal costs for retailers have on channel dynamics. Fang et al. (2009) analyze supply chain coordination for a two-stage, serial supply chain with uncertain demand, where the retailer has the option to introduce a store brand at a price of her choosing. They find that traditional coordinating contracts do not coordinate the chain when the national brand is more costly than the store brand. Groznik and Heese (2010a) analyze price commitment as a way for manufacturers to prevent store brand introduction. Groznik and Heese (2010b) also study store brand introduction dynamics under retail competition.

The use of private label is often associated with a product replacement in the assortment. In that regard, it is similar to a vertical integration of one brand manufacturer with the retailer, or in other words, category captainship. Wang et al. (2003) analyze its effects in a setting of many manufacturers (one of which the category captain) and a retailer that sells all brands. Using linear demand structures, they show that the introduction of a category captain benefits both the retailer and the category captain. Their insights on category captainship are similar to those in Raju et al. (1995) on store brands, and they show that the two are in fact substitutes. Kurtuluş and Toktay (2011) analyze category management practices under limited shelf space. They find that a retailer will benefit from category captainship provided that the profit sharing terms with the category captain are sufficiently advantageous, and that the non-captain may also find it beneficial sometimes. Here, we also show that the coalition of retailer and private label manufacturer always benefits, the other manufacturer loses, but our study mostly focuses on the effect on total supply chain profit, which may be reduced.

The focus of our analysis, in contrast with earlier papers, is to understand the impact of private label introduction on supply chain efficiency, which is a missing element in the literature. In this respect, our paper is close to the research on supply chain coordination. Spengler (1950) discusses how decentralized pricing decisions lead to loss of supply chain profits, the so-called
double marginalization. Lariviere and Porteus (2001) and Cachon and Lariviere (2001) show that double marginalization prevails even when retail prices are exogenous (in the sense that the total quantity sold on the market is smaller than the optimal quantity) under demand uncertainty. Perakis and Roels (2007) provide bounds on the resulting inefficiency. While most of these papers focus on a single retailer single manufacturer setting, Martínez-de-Albéniz and Roels (2011) analyze double marginalization with multiple retailers and a shared, limited shelf space.

3 Model and Efficiency Results

In this section, we present our main model, that we use for evaluating the impact of a private label introduction on the supply chain. We differentiate the case of having the private label replace one of the brands, or add it to the existing assortment. We provide here the analysis for linear price-demand relationships, and extend it in §4 to non-linear ones.

3.1 Benchmark with No Private Label

Consider initially a retailer that sells two products in the same category (this will be later relaxed to \( n \) products). These two products are supplied by two independent manufacturers, denoted M1 and M2. In this benchmark scenario, denoted B, there are three different parties in the supply chain, M1, M2 and the retailer, denoted R. M1 and M2 produce their products at costs \( c_1, c_2 \) and sell them to R at wholesale prices equal to \( w_1, w_2 \), respectively. R then chooses retail prices \( p_1, p_2 \) for them, or equivalently, decides on the quantities \( q_1, q_2 \) to be sold. Of course, since the players are independent, each member tries to maximize its own profit without cooperating with any of the other channel members.

We analyze this sequential decision process framing it as a non-cooperative game à la Stackelberg, where the manufacturers are leaders or first-movers, and the retailer is follower or second-mover. Our objective is to characterize the Nash equilibrium of the game, where each manufacturer has no incentive to unilaterally change its price given its competitor’s price. The analysis of the benchmark B is similar to the manufacturer-Stackelberg model in Choi (1991). This type of sequence of events (in comparison with alternatives where the retailer may be the leader) is appropriate in product categories with strong manufacturers, often category leaders, e.g., Coca-Cola and Pepsi Cola in soda or Danone and Nestlé in yoghurts. Each brand manufacturer will then choose its wholesale pricing \( w_i \) taking into account the retailer’s response function, and on the wholesale price of the competitor’s brand.
The demand model. We first model the price-demand relationship as a linear function, although we explore more general demand functions in §4. Specifically, we assume that the sales price of product \( i \), denoted \( p_i \), depends both on the quantity to be sold of the product \( q_i \), and on the quantity to be sold competitor’s product \( q_{-i} \). This linear specification is widely used in the literature, e.g., McGuire and Staelin (1983), Choi (1991) or Raju et al. (1995), although often the formulation is to describe quantity \( q_i \) as a function of \( p_i, p_{-i} \) (the two are equivalent, see below). It can be thought of the linearized version around equilibrium of any demand function. For \( i = 1, 2 \), \( q_1, q_2 \geq 0 \),

\[
p_i(q_1, q_2) = a_i - b_i q_i - sq_{-i} \tag{1}
\]

This linear specification depends on three parameters. \( a_i \) is the maximum retail price that the retailer could charge for product \( i \), which corresponds to a very low sales target, i.e., \( q_1 = q_2 = 0 \). \( b_i \) captures the sensitivity of retail price with respect to one’s sales quantity, keeping the sales of the other product identical. We set \( b_i > 0 \) in order to guarantee that the retailer never sells an infinite amount of the product. Both \( a_i, b_i \) are directly related to the scale of the market for product \( i \). Indeed, if \( q_{-i} = 0 \), then the retailer will never choose to sell more than \( \frac{a_i - c_i}{b_i} \) units of product \( i \), because that would lead to negative prices and losses. Also, \( s \geq 0 \) is the cross-quantity sensitivity, which determines the substitutability between products: for the same quantity \( q_{-i} \), the retail price that needs to be chosen to sell \( q_i \) is reduced as \( s \) increases. In this sense, the products are substitutes, which is exactly the type of strategic interaction that we want to model. Furthermore, we assume that \( s < \min \{b_1, b_2\} \) and we can note that the cross-quantity sensitivity between products is identical. Indeed, this requirement is necessary to make the model consistent with standard price-quantity functions. Indeed, since there is a one-to-one mapping between \( (p_1, p_2) \) and \( (q_1, q_2) \), we can transform this quantity-dependent price into the traditional price-dependent sales quantity:

\[
q_i(p_1, p_2) = \alpha_i - \beta_i p_i - \theta(p_i - p_{-i}) \tag{2}
\]

where \( \alpha_i = \frac{b_{-i} a_i - s a_{-i}}{b_1 b_2 - s^2} \geq 0 \), \( \beta_i = \frac{b_{-i} - s}{b_1 b_2 - s^2} \geq 0 \) and \( \theta = \frac{s}{b_1 b_2 - s^2} \geq 0 \). Hence, one can see that having a symmetric cross-quantity sensitivity implies that, given \( p_1, p_2 \), the number of customers “leaving” M1 for M2, \( \theta(p_1 - p_2) \), is equal to the number of customers switching to M2 from M1, \( -\theta(p_2 - p_1) \). Also, observe that \( s \leq \min \{b_1, b_2\} \) is necessary to ensure that the standard price sensitivity with respect to \( p_i \) keeping the price gap \( p_i - p_{-i} \) constant, denoted \( \beta_i \), is positive.
Retailer’s choice of quantities. We are now ready to define the retailer’s profit function as
\[
\Pi_R = \max_{q_1, q_2 \geq 0} \left( p_i(q_1, q_2) - w_1 \right) q_1 + \left( p_2(q_1, q_2) - w_2 \right) q_2
\]  
(3)
and \( q_{1i}^{b,r}(w_1, w_2), q_{2i}^{b,r}(w_1, w_2) \) be the unique maximizers of (3). The resulting profits of the manufacturers are
\[
\Pi_{M1} = (w_1 - c_1)q_{1i}^{b,r}(w_1, w_2) \quad \text{and} \quad \Pi_{M2} = (w_2 - c_2)q_{2i}^{b,r}(w_1, w_2).
\]  
(4)

One can observe that these profit functions are quadratic concave, which implies that it is possible to calculate analytically (1) the retailer’s best-response strategy \( q_i^{b,r}(w_1, w_2) \) to any \( w_1, w_2 \); (2) the best-response function \( w_i^{b,r} \) to the competitor’s wholesale price \( w_{-i} \), for each manufacturer M1, M2; and (3) the equilibrium wholesale prices \( w_1^B \) and \( w_2^B \), quantities \( q_i^B = q_i^{b,r}(w_1^B, w_2^B) \) and \( q_2^B = q_2^{b,r}(w_1^B, w_2^B) \), the corresponding retail prices of each product, and the profits of each firm.

Wholesale price equilibrium. In order to characterize the equilibrium, we first need to determine the best response of the retailer when the manufacturers quote \( w_1, w_2 \). Maximizing the quadratic objective in (3) yields
\[
q_i^{b,r} = \frac{c_i}{2} - \frac{\theta}{2} w_i - \frac{\theta}{2} (w_i - w_{-i}) = \frac{1}{2} q_i(w_1, w_2).
\]  
(5)
if this quantity is positive for \( i = 1, 2 \) (i.e., when the solution is interior). As one could expect, the quantity \( q_i^{b,r} \) chosen by the retailer to be sold is linearly decreasing in \( w_i \), and increasing in \( w_{-i} \). It is in fact exactly equal to half of the quantity that would be sold if the retail price was set equal to the quoted wholesale price. The corresponding retail prices are \( p_i(q_1^{b,r}, q_2^{b,r}) = \frac{a_i + w_i}{2} \), i.e., the retail price of \( i \) only depends on the wholesale price \( w_i \) and its maximum possible retail price \( a_i \). If one of the quantities in (5) is negative, say of product \( i \), then the retailer’s problem achieves a maximum with \( q_i = 0 \) and \( q_{-i} = \frac{a_{-i} - w_{-i}}{2b_{-i}} \). This can be avoided by setting \( w_i \) sufficiently low, which will occur in equilibrium whenever \( q_i^{b,r}(c_1, c_2) > 0, i = 1, 2 \). In order to avoid such trivial solutions where one manufacturer is not competitive, we assume that
\[
s \leq \min \left\{ \frac{b_2(a_1 - c_1)}{a_2 - c_2}, \frac{b_1(a_2 - c_2)}{a_1 - c_1} \right\}.
\]
Equation (5) allows us to derive the optimal best-response function of a manufacturer:
\[
w_i^{b,r}(w_{-i}) = \frac{a_i + c_i}{2} - \frac{s(a_{-i} - w_{-i})}{2b_{-i}}
\]  
(6)
Note that this wholesale price is always larger than \( c_i \) whenever the quantity allocated to \( i \) is positive (this is the case when \( q_i^{b,r}(c_1, c_2) > 0, i = 1, 2 \) which we assume to avoid
trivial equilibria). Hence, one can see that the wholesale price $w_i^{b,r}$ quoted by a manufacturer is increasing in its own cost $c_i$ and most importantly, increasing in the competitor’s wholesale price $w_{-i}$, although any increase in $w_{-i}$ results in a smaller increase in $w_i^{b,r}$ (less than half of it). This important feature implies that the manufacturer pricing game must have an equilibrium, and that this equilibrium is unique. The equilibrium is defined by wholesale prices

$$w_i^B - c_i = \frac{(2b_1b_2 - s^2)(a_i - c_i) - sb_1(a_{-i} - c_{-i})}{4b_1b_2 - s^2}. \quad (7)$$

We thus recover the results of Choi (1991), Equation (2.8). Moreover, one can verify that when $s = 0$, we obtain the standard double marginalization result with one firm, i.e., $w_i^B - c_i = \frac{a_i - c_i}{2}$. Finally, note that $w_i^B \geq c_i$ in order to guarantee that demand for product $i$ is non-negative.

3.2 Replacing a Brand by a Private Label

Consider now how the situation changes if one of the brands is replaced by a private label. We denote this scenario as PL-R (private label with replacement). This is probably the predominant choice when introducing a store brand: due to limited shelf space, the retailer removes one of the manufacturer brands from the assortment and incorporates the private label instead. For instance, this has been the path followed by Mercadona when introducing private label items, see Expansión (2009).

Characteristics of the private label. The private label product differs from the manufacturer brand in many aspects. First, the retailer typically has full control over product design, manufacturing, logistics and merchandizing decisions. Usually, private label products are produced by brand manufacturers (e.g., Friesland-Campina, a Dutch company, is organized in manufacturer brand and private label divisions), private label manufacturers (e.g., Cott Corporation, Canada) or by vertically integrated retailers (one example would be Aldi Nord in Germany, see Mitchell and Sachon (2005) or Mercadona and its “interproveedores” in Spain). In any of these cases, there is a virtual vertical integration between producer and retailer, since the retailer essentially paying the good at variable cost (it is the purchasing power of the retailer that drives this cooperation). Second, operating a private label allows to reduce distribution and store costs because of the better integration of manufacturer and retailer logistics (decrease in $c_i$). Third, the private label can get access to better displays than the brand, or to lower or higher marketing expenditures, hence increasing or decreasing the potential demand compared to a manufacturer brand (change in $a_i$). Finally, private labels have been identified as an effec-
tive lever for fostering consumers’ loyalty; as result, the sensitivity of consumers to the price of
the private label is lower (decrease in $b_i$).

While we examine the impact of all these factors on equilibrium, we start considering that
all the problem parameters do not change, and later study how they influence the equilibrium
outcome. The main change in the analysis is due to the change in how decisions are taken.
Specifically, the private label product is now integrated within the retailer’s organization. As
a result, the retailer receives a wholesale price for this product equal to its production cost. In
other words, in this scenario, there are only two players to consider, M1 and the coalition of
M2 and R, with $w_2 = c_2$. Compared to the benchmark scenario, the game in this scenario is
simply a sequential game where first the brand manufacturer sets its wholesale price and then
the retailer sets retail prices for both products. This setting is similar to Wang et al. (2003).

Properties of the equilibrium outcome with private label. Under the PL-R scenario,
the retailer’s problem is identical to the B scenario. In contrast, the wholesale price equilibrium
is characterized by $w_2^{PL-R} = c_2$ and

$$w_1^{PL-R} = 1b.r. (c_2) = \frac{a_1 + c_1}{2} - \frac{s(a_2 - c_2)}{2b_2} \leq w_1^B$$

(8)

This inequality is true since $w_2^B \geq w_2^{PL-R} = c_2$ and $1b.r. (w_2)$ is increasing in $w_2$. In other words,
the introduction of a private label increases the price pressure on the first manufacturer, M1.
Thus in equilibrium M1’s wholesale price is lower. The impact on retailer and manufacturers’
profits can be deduced from this: M1 is worse off after the private label is introduced, and the
coalition of M2 and the retailer is better off.

Indeed, the profit of manufacturer M1 can be written as a function of the equilibrium value
$w_2$:

$$\Pi_{M1}(w_2) = \max_{w_1} (w_1 - c_1) q_1^{b.r.}(w_1, w_2).$$

(9)

The same can be done for the profit of manufacturer M2 and retailer R:

$$\Pi_{M2}(w_2) = (w_2 - c_2) q_2^{b.r.}(w_1^{b.r.}(w_2), w_2)$$

(10)

$$\Pi_R(w_2) = \max_{q_1, q_2 \geq 0} \left[ p_1(q_1, q_2) - w_1^{b.r.}(w_2) \right] q_1 + \left[ p_2(q_1, q_2) - w_2 \right] q_2$$

(11)

Clearly $\Pi_{M1}$ is increasing in $w_2$ (because using the envelope theorem $\frac{d\Pi_{M1}}{dw_2} = (w_1 - c_1) \frac{\partial q_1^{b.r.}}{\partial w_2} \geq 0$). Also, $\Pi_{M2} + \Pi_R \leq \max_{q_1, q_2 \geq 0} \left[ p_1(q_1, q_2) - w_1^{b.r.}(w_2) \right] q_1 + \left[ p_2(q_1, q_2) - c_2 \right] q_2$, which is
decreasing in $w_2 \geq c_2$. Hence, since $w_2^B \geq w_2^{PL-R}$, the monotonicity results above yield the
following proposition.
Proposition 1. \( \Pi_{M1}^{PL-R} \leq \Pi_{M1}^B \) and \( \Pi_{M2}^{PL-R} + \Pi_R^{PL-R} \geq \Pi_{M2}^B + \Pi_R^B. \)

Interestingly, the equilibrium is very sensitive to the value of the cross-quantity sensitivity \( s \). We illustrate in Figure 1 the change in the equilibrium values as a function of \( s \), for symmetric products \((a_1 = a_2 = a, b_1 = b_2 = b, c_1 = c_2 = c)\). As \( s \) increases, the intensity of competition increases: \( \theta \) defined in (2) increases fast to infinity as \( s \to b \). This implies that the same price gap between products results in higher substitution of the more expensive product for the cheaper product. Intuitively, a very high value of \( s \) implies that the market is quite competitive, with low profits of the manufacturers in scenario B. As a result, introducing a private label in this region introduces minimal changes the wholesale prices and quantities in equilibrium.

One can see in the lower part of Figure 1 that retail and wholesale prices in scenario B are identical and decrease. In scenario PL-R, in contrast, the price of M1, \( w_1^{PL-R} \) is smaller than \( w_1^B \) and also decreases with \( s \); the difference \( w_1^B - w_1^{PL-R} \) first increases and then decreases with \( s \), but stays positive. Note that the difference goes to zero at \( s = 0 \) because then products are independent and the reduction of \( w_2 \) has no effect on \( w_1 \), and at \( s = 1 \) because then the retailer decides to sell only the product with lower wholesale price, which drives both \( w_1 \) and \( w_2 \) to cost both in B and PL-R. The upper left part of the Figure 1 shows the corresponding quantities. Interestingly, under B, the equilibrium sales is non-monotonic in \( s \). Indeed, as \( s \) increases, competition between manufacturers drives prices lower but because \( \beta_i \) increases with \( s \), the equilibrium sales quantity \( \frac{1}{2}(\alpha_i - \beta_i w^B) \) decreases as well. As \( s \) becomes sufficiently large, \( \alpha_i \) starts increasing faster, leading to an increase in the sales quantity. In contrast, under PL-R, quantities are decreasing in \( s \) because the reduction in wholesale prices dominates the effect of \( s \) on \( \alpha_i \). Finally, the effect on profits is depicted in the upper right part of Figure 1. Manufacturer profits in B are decreasing in \( s \), because the decrease in wholesale prices is the dominant factor. In contrast, retailer profits for scenario B behave as quantities: they are decreasing and then increasing in \( s \). This leads to profits of M1 being decreasing in \( s \), while the combined profits of M2 and R are first decreasing and then increasing. Comparing PL-R with B, we can observe that \( \Pi_{M1}^{PL-R} < \Pi_{M1}^B \), but the gap of these two profits is increasing and then decreasing with \( s \), since as we pointed out above, both at the extremes \( s = 0, 1 \), the wholesale prices and quantities are the same. On the other hand, the joint profits of retailer and M2 increase under scenario PL-R, although again this increase is minimal when \( s \) is large.

From the results above, we can highlight three main conclusions. First, the introduction of a private label will change the pricing dynamics in the channel. Indeed, this is equivalent to removing the double marginalization on the product, i.e., reducing the wholesale price \( w_2 \) to the true cost \( c_2 \). The strategic effect of this wholesale price reduction is a reduction of the
wholesale price of the other manufacturer. Second, the private label will capture higher sales than before, through a larger demand due to lower retail price, but also through substitution of the branded product of M1 with the private label. The first product thus will reduce its sales. Third, the retailer is always better off by joining forces with a manufacturer through the introduction of a private label in terms of profits, which would explain why so many retailers are considering this type of action. In contrast, the other manufacturer in the category, M1, obtains lower profits. These three conclusions are prevalent in the literature, see Raju et al. (1995) or Wang et al. (2003) for instance.

Figure 1: On top, equilibrium quantities (left) and profits of M1 and the coalition M2 plus R (right), under B and PL-R, for different values of $s$; at the bottom, retail and wholesale prices. We use $a_1 = a_2 = 1$, $b_1 = b_2 = 1$ and $c_1 = c_2 = 0.2$. 
Impact on supply chain efficiency. While retailer and M2 are doing better with the private label, M1 is doing worse. An important question is whether the introduction of the private label is positive for the entire supply chain. Indeed, if the profit improvement for the retailer is at the expense of supply chain efficiency, value is being destroyed, and hence it would be possible to find a Pareto-improving solution. Namely, it would be better for everyone not to introduce the private label, and instead pass side-payments from manufacturers to the retailer. Figure 1 suggests that for $s$ close to zero, the profit reduction for M1 is minimal, and hence the supply chain benefits. However, for large $s$, this is no longer obvious.

For this purpose, we define the supply chain profit as $\Pi_{SC} := \Pi_{M1} + \Pi_{M2} + \Pi_{R} = (p_1(q_1, q_2) - c_1)q_1 + (p_2(q_1, q_2) - c_2)q_2$. To build some intuition, the combined profit of manufacturers and retailer will always be smaller than that of a centralized chain, which is called the first-best. It is achieved by setting $(q_1^*, q_2^*)$ that maximize $\Pi_{SC}$. In other words, $q_1^* = q_1^{br}(c_1, c_2)$ and $q_2^* = q_2^{br}(c_1, c_2)$.

In scenarios B and PL-R, the equilibrium wholesale prices are higher than the cost. As a result, $q_1, q_2$ are distorted as compared to the first-best:

- Total unit sales are lower than in the first-best;
- The mix of sales is different compared to that of the first-best, i.e., each individual product might sell more or less than in the first-best scenario.

These two effects create inefficiency for the supply chain: double marginalization in a two-product environment. Interestingly, the first effect (on total sales) is likely to be more important in B compared to PL-R, because prices are lower in PL-R. On the other hand, the second effect may actually be stronger in PL-R. Thus the overall efficiency of the B and PL-R equilibria will depend on the strength of this second effect. It ultimately depends on the retail price gap for each of the scenarios in equilibrium. For an illustration, we define $\Delta := p_1 - p_2$ for symmetric demands ($a_1 = a_2 = 1, b_1 = b_2 = 1$). In B,

$$\Delta^B := p_1^B - p_2^B = \frac{(2 - s)}{2(4 - s^2)}(c_1 - c_2).$$

Similarly,

$$\Delta^{PL-R} := p_1^{PL-R} - p_2^{PL-R} = \frac{1}{4} + \frac{1}{4}c_1 - \frac{2 - s}{4}c_2.$$ 

Furthermore, observe that the price gap in the first-best is

$$\Delta^* := p_1^* - p_2^* = \frac{1}{2}(c_1 - c_2).$$

From these expressions, we observe that the price gap might indeed be higher under PL-R compared to B. For example, when the two manufacturers have the same cost, in the B
equilibrium and in the first-best, the price gap is zero. In contrast, the price gap is positive under PL-R equilibrium.

Since we have explicit expressions of \( \Pi_{SC}^B \) and \( \Pi_{SC}^{PL-R} \), we can compare the efficiency of the two scenarios. \( \Pi_{SC}^{PL-R} - \Pi_{SC}^B \) is an intricate function of the parameters. For the special case of symmetric demands and \( c_1 = c_2 = c \), the expression simplifies to:

\[
\Pi_{SC}^{PL-R} - \Pi_{SC}^B = \frac{(a - c)^2(b - s)(4b^2 - 4bs - s^2)}{16b(b + s)(2b - s)^2}
\]

(12)

Figure 2: Total supply chain profits for the two scenarios, together with the first-best profit \( \Pi_{SC}^* \). We use \( a_1 = a_2 = 1, b_1 = b_2 = 1 \) and \( c_1 = c_2 = 0.2 \).

Under symmetric costs, Figure 2 illustrates the value of \( \Pi_{SC}^B, \Pi_{SC}^{PL-R} \) and \( \Pi_{SC}^* \) as a function of the cross-sensitivity \( s \). We observe that \( \Pi_{SC}^{PL-R} \geq \Pi_{SC}^B \) if and only if \( s \) is lower than a certain threshold. This is intuitive: for low substitution levels, the scenario B results in low efficiency since the manufacturers quote prices that are too high, while the scenario PL-R is effective in reducing M1’s wholesale price and hence yields higher efficiency. However, when the substitution level is high, the asymmetry introduced in PL-R does not reduce wholesale prices much and reduces the total margin achieved. In the following proposition, we calculate the threshold explicitly, and determine the general conditions for comparing the supply chain performance in PL-R and B.

**Proposition 2.** There exists \( \bar{s} \in (0, 1] \) such that \( \Pi_{SC}^{PL-R} \geq \Pi_{SC}^B \) if and only if \( s \leq \bar{s} \). When \( a_1 - c_1 = a_2 - c_2 \) and \( b_1 = b_2 = b \), \( \bar{s} = 2(\sqrt{2} - 1)b \approx 0.83b \).
The proposition implies that there exists a threshold that determine the region in which the private label may be detrimental to the supply chain. In terms of efficiency, PL-R is beneficial compared to B only when product substitutability is not too high. This result indicates that in this range the retailer not only has a personal interest in introducing a private label (because it improves its profits), but also for the sake of supply chain efficiency.

More generally, when $a_2 \neq a_2$, $b_1 \neq b_2$ or $c_1 \neq c_2$, Figure 3 depicts the value of the threshold $\bar{s}$ as a function of the demand parameters. Interestingly, the value of $\bar{s}$ increases as $a_2$ or $b_1$ are higher, and $a_1$ or $b_2$ lower. In other words, the region of supply chain efficiency gains is larger when the private label has access to a larger market (or equivalently when the price to achieve a certain sales target is higher), because then the price pressure on M1 is effective in reducing $p_1$, and at the same time customers switch from the less profitable product 1 into the more profitable private label. This is also true when $b_2$, the sensitivity of $p_2$ to $q_2$, decreases, because then again the private label is more attractive than product 1; in particular, if $b_1$ is slightly larger than $b_2$, then $\Pi_{SC}^{PL-R} > \Pi_{SC}^{R}$ for all the range of acceptable $s$ ($s < \min\{b_1, b_2\}$). This implies that, not only the retailer would prefer to replace the ‘best’ product in the assortment by a private label, out of all possible products, but this is also the choice that has better of chances of increasing supply chain efficiency.

It is worth comparing this threshold with the parameter estimates from the empirical literature. Cotterill et al. (2000) provide explicit values of the demand parameters for several categories. While they consider a logarithmic demand model, measure market share instead of unit sales and find non-symmetric cross-sensitivities, we still can use these demand parameters in our model. Indeed, consider the pasta category for instance. Reducing the price of the
private label by 1 cent amounts to a percentage reduction of 1.5%, which results in a quantity increase of about 5.4% for the private label; similarly, the reduction of 1 cent in national brand price yields a 1.0% reduction in price and hence a decrease of 3.0% in private label sales. Similarly, the impact of a 1 cent reduction on the private label is to decrease national brand sales by 0.6%, while a 1 cent reduction on the national brand increases sales by 1.5%. Given that the national brand has a market share of 81%, this suggests that the values to use in our model are $\beta_1 + \theta = 0.0119, \beta_2 + \theta = 0.0102$ and $\theta$ between 0.0050 and 0.0056, which corresponds to $b_1 = 109, b_2 = 127$ and $s = 57$. For these parameters and assuming $a_1 = a_2$, we find that supply chain efficiency is indeed higher with the private label than with two national brands. However, the value of $s$ is not too far from the point where the private label starts being detrimental, which would occur if the price gap was larger.

The role of different characteristics. As we mentioned above, private labels change the nature of demand characteristics and costs. Typically, they decrease self-quantity sensitivity $b_2$, decrease cost $c_2$ and may increase or decrease the price required to achieve a certain sales quantity $a_2$ depending on how the retailer displays the private label. That is $a_2^{PL-R}, b_2^{PL-R}, c_2^{PL-R}$ are different from $a_2, b_2, c_2$ used in scenario B. It is simple to characterize the influence of the parameters on the supply chain profits, as done in the following proposition.

Proposition 3. If $a_2^{PL-R}$ increases or $b_2^{PL-R}, c_2^{PL-R}$ decrease, then the region over which $\Pi_{SC}^{PL-R} \geq \Pi_{SC}^{B}$ becomes larger.

3.3 Adding a Private Label

When shelf space is ample, the retailer does not need to remove one of the existing brands from the assortment. We consider here the impact of an added private label. We denote this scenario as PL-A (private label with addition). While this is a less common practice than the replacement, it has interesting implications, as it increases supply chain efficiency in most situations. The reason for this is that adding a new product to the assortment typically increases the market potential. As a result, the first-best profit with a third product is higher than $\Pi_{SC}^{*}$ in PL-R. Hence, as $\Pi_{SC}^{PL-A}$ approaches this first-best profit for large $b$, it becomes higher than $\Pi_{SC}^{B}$ in this range as well.

Demand model with three products. With the addition of a private label, the demand specification needs to include the influence of the third product on the other products. In order
to keep the three-product model with Equation (1), we let
\[ p_i(q_1, q_2, q_3) = a_i - b_i q_i - \sum_{j \neq i} s_{ij} q_j \] 

(13)

Hence, \( p_i(q_1, q_2, 0) \) corresponds to the retail price determined without a private label. From §3.1, we have that \( s_{12} = s_{21} \). We similarly impose that \( s_{13} = s_{31} \) and \( s_{23} = s_{32} \). Sayman et al. (2002) suggest that it is likely that \( s_{12} < s_{13}, s_{23} \) for private labels that are positioned close to the existing leading brands.

Equation (13) yields
\[ q_i(p_1, p_2, p_3) = \tilde{\alpha}_i - \tilde{\beta}_i p_i - \sum_{j \neq i} \tilde{\theta}_{ij} (p_i - p_j) \] 

(14)

Denoting \( X = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} \) and letting \( M = \begin{pmatrix} b_1 & s_{12} & s_{13} \\ s_{12} & b_2 & s_{23} \\ s_{13} & s_{23} & b_3 \end{pmatrix} \), we can write \( p = a - M q \) and hence the parameters in (14) are given by \( \tilde{\alpha} = M^{-1} a \) and
\[
\begin{pmatrix}
\tilde{\beta}_1 + \tilde{\theta}_{12} + \tilde{\theta}_{13} & -\tilde{\theta}_{12} & -\tilde{\theta}_{13} \\
-\tilde{\theta}_{12} & \tilde{\beta}_2 + \tilde{\theta}_{12} + \tilde{\theta}_{23} & -\tilde{\theta}_{23} \\
-\tilde{\theta}_{13} & -\tilde{\theta}_{23} & \tilde{\beta}_3 + \tilde{\theta}_{13} + \tilde{\theta}_{23}
\end{pmatrix} = M^{-1},
\]

This model is the natural extension of the two-product model, and has been used by Raju et al. (1995) and Sayman et al. (2002) among others. It is worth noting that, in order to preserve the same parameters \( a_1, a_2, b_1, b_2, s_{12} \) in (1) and (13), the parameters \( \tilde{\alpha}, \tilde{\beta}, \tilde{\theta} \) may be different from the ones used in (2). For instance, in Raju et al. (1995) or Sayman et al. (2002), a two- and three-product demand model is used, where the three-product demand is scaled so that \( \sum_i q_i(0, 0, 0) = 1 \); besides that scaling factor (which does not change the equilibrium values, only the profits achieved), \( \tilde{\alpha}_i = \alpha_i, \tilde{\beta}_i = \beta_i \) and \( \tilde{\theta}_{12} = \theta/2 \). Such demand specification unfortunately does not preserve the price-demand relationship in general. In contrast, ours is the natural extension where \( p_i(q_1, q_2, 0) \) in the three-product model is equal to \( p_i(q_1, q_2) \) in the two-product one.

We can again derive the prices associated with a certain level of sales. As in the two-product model, we require that \( M \) is invertible, which is true when it is a diagonally-dominant symmetric matrix: \( b_i \geq \sum_{j \neq i} s_{ij} \). \footnote{When \( s_{12} = s_{13} = s_{23} = s \) a weaker condition is that \( s < \min\{b_1, b_2, b_3\} \).} this condition, \( M^{-1} \) is well defined and definite positive.

The retailer’s problem is to maximize \( q^t (a - M q) - q^t w \), where \( X^t \) is the transpose of \( X \). The optimal quantities are thus \( q^{br}(w) = \frac{1}{2} M^{-1} (a - w) = \frac{1}{2} (\tilde{\alpha} - M^{-1} w) \).
The wholesale prices are such that \( w_3 = c_3 \) (the private label) and \( w_i, i = 1, 2 \), are the equilibrium in the pricing game. Manufacturer \( i \) maximizes its profit, half of \( (w_i - c_i)q_i(w) \). The best-response function of \( i = 1, 2 \), is hence

\[
\begin{align*}
 w_{b,r}(w_{-i}) &= \frac{\alpha_i + \sum_j \theta_{ij} w_j}{\beta_i + \sum_j \theta_{ij} w_j + 2} + c_i \\
\end{align*}
\]

Again, because the best-responses are increasing with slope less than one, there is a unique equilibrium.

**Impact on profits and supply chain efficiency.** The addition of one brand changes the profits of retailer and manufacturers. The next proposition highlights similar dynamics to Proposition 1.

**Proposition 4.** For \( i = 1, 2 \), \( \Pi_{PL}^{PL-A} \leq \Pi_{PL}^{B} \), and \( \Pi_{PL}^{PL-A} \geq \Pi_{PL}^{B} \).

The impact on the supply chain can also be studied. In comparison with Proposition 2, we find that \( \Pi_{SC}^{PL-A} \) is typically higher than \( \Pi_{SC}^{B} \) when \( s \leq \bar{s}_{low} \) or \( s \geq \bar{s}_{high} \). Figure 4 depicts an example of this result. Furthermore, depending on the parameters, it may be better to replace or to add a private label, although for low substitution parameters \( s_{12}, s_{13}, s_{23} \), PL-A leads to higher supply chain efficiency compared to PL-R, because in this range the increase in market size outweighs the substitution of higher profit products by lower profit ones.

## 4 Generalized Demand and Number of Manufacturers

We focused so far on the analysis of linear demand models. The linearity assumption was chosen for two reasons: first, it allows an analytical study of equilibrium prices, quantities and profits, which generates insights; second, it approximates locally real demands, which implies that the conclusions reached above apply provided that the approximation is accurate in the vicinity of the equilibrium. In this section, we generalize the demand model and extend the analysis to \( n \) manufacturers. We characterize the conditions under which the results of §3 continue to apply, focusing on the comparison between B and PL-R scenarios. Note that the analysis of general demand functions quickly becomes intractable even in a duopoly, as pointed out by Choi (1991). Because of this, we focus on identifying the structural conditions that preserve the problem’s structure.

For this purpose, with \( n \) products we consider a general demand structure \( p_i(q) \) that replaces Equation (1). If products are substitutes, then \( p_i \) is decreasing in \( q_j \), for \( j = 1, \ldots, n \). The retailer’s
profit can be written as $\Pi_R(w) = \max_{q \geq 0} q^t(p(q) - w)$. In order to preserve the structure of the analysis, we need first to guarantee that the retailer’s problem has a well-behaved solution: $q^t p(q)$ needs to be strictly concave.\(^3\) Sufficient conditions for this have been provided recently in the literature, see Song and Xue (2007) in a context of inventory planning with substitution. This condition applies for instance to the multinomial logit (MNL) choice model, where sales are equal to 

$$e^{\mu_i(\lambda_i - r_i)} \left( 1 + \sum_{j=1}^n e^{\mu_j(\lambda_j - r_j)} \right),$$

and hence $p_i = \lambda_i + \frac{1}{\mu_i} \left( \ln \left( 1 - \sum_{j=1}^n q_j \right) - \ln(q_i) \right)$.

Hence, when $q^t p(q)$ is concave, the retailer’s choice of $q^{b,r}(w)$ is uniquely characterized by the first-order conditions:

$$p_i(q) + \sum_{k=1}^n q_k \frac{\partial p_k}{\partial q_i} = w, i = 1, \ldots, n.$$ 

(15)

Each manufacturer now faces the problem of maximizing $(w_i - c_i) q_i^{b,r}$. Given that $\frac{d\Pi_{Mi}}{dw_i}$ =

\(^3\)In fact, weaker conditions could be sufficient provided that (i) there is a unique optimal solution $q_i^{b,r}(w)$; and (ii) that this best-response is continuously differentiable in $w$.  

Figure 4: Supply chain profits for $a_1 = a_2 = a_3 = 1$, $b_1 = b_2 = 1$, $b_3 = 0.75$, $c_1 = c_2 = c_3 = 0.2$ and $s_{12} = s_{13} = s_{23} = s$ variable.
\[
\frac{\partial q_i^{b.r.}}{\partial w_i} \left( w_i - \frac{q_i^{b.r.}}{\partial w_i} - c_i \right), \text{ there is a unique local maximum when for all } w_i, \ i = 1, \ldots, n, \]
\[
\quad w_i - \frac{q_i^{b.r.}}{\partial w_i} \text{ increases in } w_i \text{ when } \frac{\partial q_i^{b.r.}}{\partial w_i} \leq 0. \tag{16}
\]

Under this condition we can define uniquely \( w_i^{b.r.}(w_{-i}) \) as a continuous function. In addition, from Cachon and Netessine (2004), a sufficient condition for the wholesale price equilibrium to be unique is that for all \( i \),
\[
\frac{dw_i^{b.r.}}{dw_j} \geq 0 \text{ and } \sum_{j \neq i} \frac{dw_i^{b.r.}}{dw_j} < 1, \tag{17}
\]
which is true in the linear demand case. These properties are also satisfied for the MNL choice model, when products have identical sensitivities, as shown below.

**Proposition 5.** When the demand follows is given by the MNL, i.e.,
\[
p_i = \lambda_i + \frac{1}{\mu_i} \left[ \ln \left( 1 - \sum_{j=1}^{n} q_j \right) - \ln(q_j) \right],
\]
then \( q^i p(q) \) is strictly concave. Furthermore when \( \mu_i = \mu \), (16)-(17) are satisfied and there is a unique equilibrium of the manufacturer’s pricing game in all scenarios B, PL-R and PL-A. In this equilibrium, quantities must satisfy \( q_0 = 1 - \sum_{k=1}^{n} q_k \),
\[
\mu(\lambda_i - c_i) - \frac{1}{q_0} - \ln \left( \frac{q_i}{q_0} \right) = \frac{1}{1 - (1 + q_0)q_i}
\]
if product \( i \) is a manufacturer brand and
\[
\mu(\lambda_i - c_i) - \frac{1}{q_0} - \ln \left( \frac{q_i}{q_0} \right) = 0
\]
if product \( i \) is a private label.

Under the conditions of strict concavity of \( q^i p(q) \) and Equations (16)-(17), the introduction of the private label that replaces product \( n \) always reduces the equilibrium wholesale price of the brands, since the equilibrium wholesale price \( w_n^B \) of the integrated product is reduced to \( w_{n-1}^{PL-R} = c_n \).

The impact on profit can also be established. Letting \( w_i^{eq}(w_n) \) be the equilibrium of the game between manufacturers \( i = 1, \ldots, n-1 \) fixing \( w_n^{eq} = w_n \), (17) directly implies that \( w_i^{eq} \) are increasing in \( w_n \). The associated quantities \( q_i^{eq}(w_n), i \neq n \), are also increasing in \( w_n \).

Denoting \( \Pi_M(w_n) = (w_i^{eq}(w_n) - c_i)q_i^{eq}(w_i^{eq}(w_n)) \) and \( \Pi_R(w_n) = \max_{q \geq 0} q^i(p(q) - w^{eq}(w_n)) \), we conclude that the profit of \( Mi, i = 1, \ldots, n-1 \) is reduced because \( \Pi_{M_i}^B = \Pi_{Mi}(w_i^B) \),
\[ \Pi_{PL-R}^{Mi} = \Pi_{Mi}(c_n) \] and, from the envelope theorem, \[ \frac{d\Pi_{Mi}(w_n)}{dw_n} = (w_{i}^{eq}(w_n) - c_i) \frac{dq_i^{eq}}{dw_n} \geq 0. \]

Similarly, the joint profit of Mn and the retailer increases since in scenario B it is at most

\[ \max_{q \geq 0} \sum_{j=1}^{n} q_j \left( p_j(q) - w_j^{eq}(w_n) \right) + q_n(p_n(q) - c_n) \] and this is decreasing in \( w_n \).

The impact of a private label introduction on supply chain profit is more difficult to evaluate. Generally, letting \( \Pi_{SC}(w_n) = \sum_{i=1}^{n} \Pi_{Mi}(w_n) + \Pi_{R}(w_n) \),

\[ \frac{d\Pi_{SC}}{dw_n} = \sum_{i=1}^{n} \left( p_i - c_i + \sum_{j=1}^{n} q_j \frac{\partial p_j}{\partial q_i} \right) \frac{dq_i^{eq}}{dw_n} = \sum_{i=1}^{n} (w_i^{eq} - c_i) \frac{dq_i^{eq}}{dw_n} \]

because \( p_i - w_i^{eq} + \sum_{j=1}^{n} q_j \frac{\partial p_j}{\partial q_i} = 0 \). Since \( \frac{dq_i^{eq}}{dw_n} \geq 0 \) for \( i = 1, \ldots, n-1 \) and \( \frac{dq_i^{eq}}{dw_n} \leq 0, \) the sign of \( \frac{d\Pi_{SC}}{dw_n} \) may be positive or negative depending on how large \( w_i^{eq} - c_i \) is. Hence, only when double marginalization is important (high \( w_i^{eq} - c_i \)), then the replacement of product \( n \) by a private label will increase supply chain profits.

Finally, we illustrate the impact of \( n \) with a numerical study based on the MNL model. Specifically, we let \( p_i = \ln \left( 1 - \sum_{j=1}^{n} q_j \right) - \ln(q_i) \) and \( c_i = 0 \) for all products. We illustrate the B and PL-R equilibrium for different values of \( n \) in Figure 5. We observe that, consistently with our results in §3, scenario PL-R leads to lower supply chain profits when scenario B is sufficiently competitive – when \( n \) is high. Furthermore, scenario PL-A increases supply chain profit always because, due to the symmetry of the demand function, the increase of sales due to the additional product dominates the loss of profits due to lower revenues associated with the private label.

5 Conclusions and Discussion

In this paper, we have presented a simple model to evaluate the effect on supply chain performance of introducing a private label. Our analytical results coincide with the existing literature: the introduction of a private label product into a product category, by replacement or addition, forces the incumbent manufacturer brand to a strategic price reduction. This reduces the rents realized by the manufacturer brand. The price reduction of the manufacturer brand is more pronounced when the substitutability between the manufacturer brand and private label product is neither too high nor too low. For high or low substitutability, the wholesale price of the brand manufacturer will not change much, because when products are independent, the private

21
Figure 5: Equilibrium value of brand manufacturer wholesale price in scenarios B, PL-R and PL-A (left), and supply chain profit variation in PL-R and PL-A compared to B (right), as a function of $n$ the number of manufacturers in scenario B.

label will not affect the other manufacturer, while when product are very strong substitutes, competition without private label already reduced prices close to the variable production cost.

One of the main conclusions derived from our model is that the supply chain might be worse off after the retailer replaces one manufacturer brand by a private label, or adds the private label to the existing assortment. Indeed, while introducing the private label reduces prices and the double marginalization effect on total sales, it also distorts the price gap between the items in the category, which might drive too many sales to the private label, thereby reducing supply chain profit. We found that, under replacement, the introduction of private label is only beneficial for low values of product substitutability $s$, and we identified the threshold $\bar{s}$ after which private label will hurt supply chain performance. This threshold increases as the private label has the advantage of lower distribution and store costs, increases the base demand for the product or reduces the customer sensitivity to price due to increased store loyalty. Under addition, the private label may hurt supply chain efficiency when $s$ is not too high nor too low.

While the basic model considers only two manufacturers and linear demand functions, we extend these insights to more general situations, with $n$ products and non-linear demands. Generally, under some regularity conditions on the demand function, the introduction of a private label reduces supply chain profit when the competition in the category is initially strong. This is the case for example when the number of players in the category is large. These conditions are satisfied for example when demand follows the multinomial logit choice model. These extensions suggest that our conclusions on supply chain efficiency are robust.
Our results allow us to formulate several recommendations for retail supply chains. First, manufacturers should realize that retailers will always find it interesting to introduce a private label and be prepared to face more competition through lower wholesale prices. They will be forced to reduce their prices to retailers, especially when substitutability is not too high nor too low. Second, retailers may want to introduce private labels even when this reduces the size of the pie, i.e., total supply chain profit. In this case, brand manufacturers are the most affected by the profit reduction. They should be proactive and provide a Pareto-improving alternative, such as lower prices or a lump-sum payment to the retailer, in return for not introducing the value-destroying private label. Finally, the option of adding a private label, despite growing the potential market due to an increased assortment, may not always be better than replacing a brand, because again too many sales may be driven to the less profitable private label.

While our model adds to the growing literature of private labels in retail, it opens a number of possible lines for future work. First, we identify the strategic impact of private label introduction on category prices. However, there are other side effects that may be as important as prices: shelf space allocations, product placement and promotional activities. These are short-term decisions that manufacturers base on what competitors do, and are likely to be influenced by the replacement of a brand manufacturer by a private label controlled by the retailer. In this respect, continuous analytical models can shed light on how these operational and marketing variables change with private labels. Second, the introduction of private labels is usually followed by a rationalization of the category, i.e., the number of items in the assortment is modified. It would be interesting to model the retailer’s optimal assortment with and without private label to evaluate the changes for all players in the category and the supply chain.

Acknowledgements

We would like to thank a senior editor and three anonymous referees for helping us improve significantly this manuscript.

References


is decreasing, first positive and then negative. Hence the third derivative

\[ N^{(3)}(u) = -120uy^2 + (144 - 360u^2)y + (96u - 120u^3) \]
is concave increasing and then decreasing. Since it is positive when $u = 0$, it is first positive and then negative. As a result, the second derivative is

$$N''(u) = (24 - 60u^2)y^2 + (144u - 120u^3)y + (-24 + 48u^2 - 30u^4)$$

is first increasing and then decreasing, and hence may be negative, positive and then negative again. Thus the first derivative

$$N'(u) = (24u - 20u^3)y^2 + (-32 + 72u^2 - 30u^4)y + (-24u + 16u^3 - 6u^5)$$

is negative when $u = 0$, decreases, then increases (it may go positive) and then decreases (it may go negative again). This implies that $N(u)$ decreases, increases and decreases again. Since it is positive when $u = 0$ and $u = 1$, it can go negative at most twice in $(0, 1)$.

Consider now the constraint imposed on $s$, that $s \leq \min\{b_2a_1, b_1a_2\}$. This is equivalent to imposing that $u^2 \leq \min\{x_1/x_2\} = \min\{y - 1/y\}$. One can check that when $u^2 = y = 1$, $N(u) \geq 0$ and $N'(u) \leq 0$, which means that $N(u) \geq 0$ for all $u \in [0, \sqrt{y}]$, and when $u^2 = 1/y \leq 1$, $N(u) \leq 0$, implying that $N(u) \geq 0$ if and only if $u$ is lower than a threshold.

When $b_1 = b_2$ and $a_1 = a_2$, $y = 1$ and the first derivative is negative and then positive. Setting (12) to zero leads to

$$(-32u + 24u^3 - 6u^5) + (16 - u^4 - u^6) = (1 - u)^2(2 + u)^2(4 - 4u - u^2) = 0,$$

so that the only solution in $(0, 1)$ is $u = 2(\sqrt{2} - 1)$. ■

Proof of Proposition 4

Proof. We can again write the equilibrium values $w_1, w_2$ as a function of $w_3$. It is clear that $\Pi_{M1}$ is increasing in $w_3$ and $\Pi_R$ is decreasing in $w_3$. Observe that setting $w_3$ sufficiently high value such that $q_3 = 0$ yields the equilibrium of scenario B. Hence, reducing $w_3$ down to $c_3$ decreases the equilibrium profits of $M1, M2$ and increases the profits of $R$. ■

Proof of Proposition 5

Proof. Let $q_0 = 1 - \sum_{k=1}^n q_k$ and $\phi(q) = q^t p(q)$. Then

$$\frac{\partial \phi}{\partial q_i \partial q_j} = -\left(\frac{\sum_{k=1}^n q_k / \mu_k}{q_0^2} + \frac{1}{\mu_i q_i} + \frac{1}{q_0} \left(\frac{1}{\mu_i} + \frac{1}{\mu_j}\right)\right).$$

Since the $n \times n$ matrix with entries equal to one is positive semi-definite, a diagonal matrix with positive diagonal is positive definite, and for all $x$, $\sum_{i,j \in [1,n]} \left(\frac{1}{\mu_i} + \frac{1}{\mu_j}\right) x_i x_j = \left(\sum_{k=1}^n \frac{x_k}{\mu_k}\right)^2 \geq 0$, then the Hessian matrix $H$ of $\phi$ is negative definite, i.e., $\phi$ is concave. This result is contained in Proposition 2 of Song and Xue (2007).
Assuming $\mu_i = \mu$, $\mathbf{q}^{b.r.}$ is thus the unique solution of:

$$\lambda_i - \frac{1}{\mu} - \sum_{k=1}^{n} \frac{g_k}{\mu q_0} - \frac{1}{\mu} \left[ \ln \left( \frac{q_i}{q_0} \right) \right] = w_i, \ i = 1, \ldots, n.$$ 

In other words,

$$q_i = q_0 e^{\mu(\lambda_i - w_i) - \frac{1}{\mu}}$$

Since $q_i$ is increasing in $q_0$, there is a unique solution such that $\sum_{k=1}^{n} q_i = 1 - q_0$. Implicit differentiation of the equations above yields $\frac{1}{q_i} \frac{\partial q_i}{\partial w_i} = \left( \frac{1}{q_0} + \frac{1}{q_0^2} \right) \frac{\partial q_0}{\partial w_i} - \mu$ and for $j \neq i \frac{1}{q_j} \frac{\partial q_j}{\partial w_i} = \left( \frac{1}{q_0} + \frac{1}{q_0^2} \right) \frac{\partial q_0}{\partial w_i}$. Noting that $\frac{\partial q_0}{\partial w_i} = - \sum_{k=1}^{n} \frac{\partial q_k}{\partial w_i} = \mu q_i - \left( \frac{1}{q_0} + \frac{1}{q_0^2} \right) (1 - q_0) \frac{\partial q_0}{\partial w_i}$, we obtain $\mu q_i = \left[ 1 + \left( \frac{1}{q_0} + \frac{1}{q_0^2} \right) (1 - q_0) \right] \frac{\partial q_0}{\partial w_i}$ and hence noting that $\frac{1}{\mu} \frac{1}{\mu q_i} \frac{\partial q_i}{\partial w_i} = (1 + q_0)q_i - 1$ and $\frac{1}{\mu q_j} \frac{\partial q_j}{\partial w_i} = (1 + q_0)q_i \geq 0$. Interestingly,

$$\frac{1}{\mu(1 + q_0)q_i} \frac{d[(1 + q_0)q_i]}{d w_i} = (1 + q_0)q_i - 1 + \frac{q_0 q_i^2}{1 + q_0} \leq (1 + q_0)(1 - q_0) - 1 + \frac{q_0^2}{1 + q_0} = - \frac{q_0^3}{1 + q_0} \leq 0.$$ 

As a result, $- \frac{1}{q_i} \frac{\partial q_i}{\partial w_i}$ is increasing and hence (16) is satisfied. $w_i^{b.r.}$ is thus uniquely defined by

$$w_i - \frac{1}{\mu [ 1 - (1 + q_0)q_i]} = c_i.$$ 

We can implicitly differentiate it with respect to $w_j, j \neq i$:

$$\frac{\partial w_i^{b.r.}}{\partial w_j} = \frac{(1 + q_0)q_i q_j}{[1 - (1 + q_0)q_i]^2} \left[ \frac{1}{1 + q_0} + \frac{q_0^2}{1 + q_0} \right] + \frac{(1 + q_0)q_i^2}{[1 - (1 + q_0)q_i]^2} \left[ \frac{1}{1 + q_0} + \frac{q_0^2}{1 + q_0} - \frac{1}{q_i} \right] \frac{\partial w_i^{b.r.}}{\partial w_j}$$

$$= \frac{q_i q_j}{[1 + q_0 - q_i]^2} \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right] + \frac{q_i^2}{[1 + q_0 - q_i]^2} \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 - \frac{1}{q_i (1 + q_0)} \right] \frac{\partial w_i^{b.r.}}{\partial w_j}$$

$$= \frac{q_i q_j}{[1 + q_0 - q_i]^2} \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right] + q_i^2 \left[ \frac{1}{q_i (1 + q_0)} - 1 - \left( \frac{q_0}{1 + q_0} \right)^2 \right] \frac{\partial w_i^{b.r.}}{\partial w_j}$$

$$\geq 0$$

and hence
\[
\sum_{j \neq i} \frac{d w^k_r}{d w^i_j} = \frac{q_i (1 - q_0 - q_i) \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right]}{\left( \frac{1}{1 + q_0} - q_i \right)^2 + q_i^2 \left[ \frac{1}{q_i (1 + q_0)} - 1 - \left( \frac{q_0}{1 + q_0} \right)^2 \right]} \\
= \frac{q_i \left( 1 - \frac{q_i}{1 - q_0} \right) \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right]}{(1 - q_0)^2 \left( \frac{1}{1 - q_0} - \frac{q_i}{1 - q_0} \right)^2 + \frac{q_i}{1 - q_0} \left( 1 - \frac{q_i}{1 - q_0} \right) \left( \frac{q_i}{1 - q_0} \right)^2 \left( 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right)} \\
\leq 1
\]

because for \( x \in [0, 1] \),
\[
\frac{x (1 - x) \left[ 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right]}{(1 - q_0)^2 \left( \frac{1}{1 - q_0} - x \right)^2 + \frac{x}{1 - q_0} - x^2 \left( 1 + \left( \frac{q_0}{1 + q_0} \right)^2 \right)} \leq 1.
\]

This proves (17) and completes the proof. \( \blacksquare \)